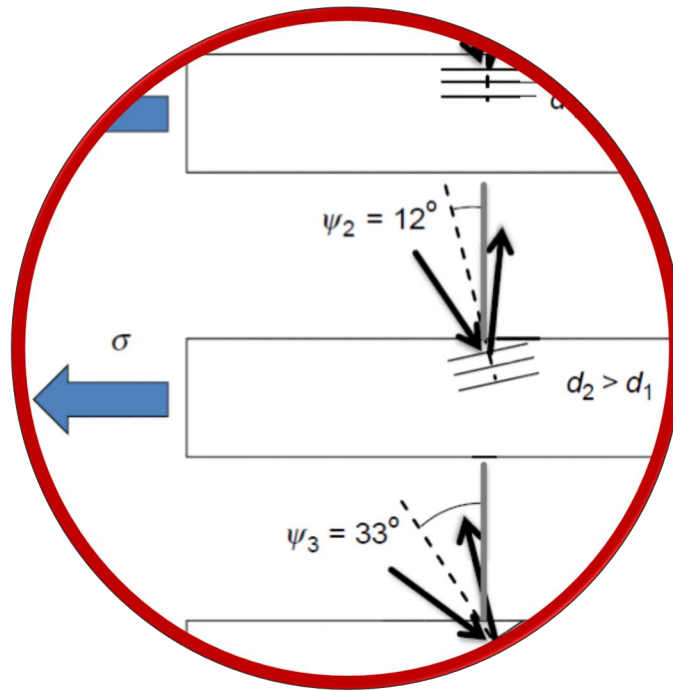


## 2. Microstructure of materials



# The Periodic Table

Elements in each column:  
Similar **valence** electron structure

Metal  
 Nonmetal  
 Intermediate

IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIII			IB	IIB	IIIA	IVA	VA	VIA	VIIA	0
1 H	2 He											3 B	4 C	5 N	6 O	7 F	8 Ne
3 Li	4 Be											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
11 Na	12 Mg											31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	Rare earth series	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	Acti-nide series	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds								

give up 1e<sup>-</sup>  
give up 2e<sup>-</sup>  
give up 3e<sup>-</sup>

accept 2e<sup>-</sup>  
accept 1e<sup>-</sup>  
inert gases

Electropositive elements:  
Readily give up electrons  
to become + ions.

Electronegative elements:  
Readily acquire electrons  
to become - ions.

# Electronegativity

- Ranges from 0.7 to 4.0,
- Large values: tendency to acquire electrons.

IA																		0
H																		He
2.1	IIA											IIIA	IVA	VA	VIA	VIIA		-
Li	Be											B	C	N	O	F		Ne
1.0	1.5											2.0	2.5	3.0	3.5	4.0		-
Na	Mg																	Ar
0.9	1.2																	-
		IIIB	IVB	VB	VIB	VIIIB	VIII			IB	IIB							
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br		Kr
0.8	1.0	1.3	1.5	1.6	1.6	1.5	1.8	1.8	1.8	1.9	1.6	1.6	1.8	2.0	2.4	2.8		-
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I		Xe
0.8	1.0	1.2	1.4	1.6	1.8	1.9	2.2	2.2	2.2	1.9	1.7	1.7	1.8	1.9	2.1	2.5		-
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At		Rn
0.7	0.9	1.1-1.2	1.3	1.5	1.7	1.9	2.2	2.2	2.2	2.4	1.9	1.8	1.8	1.9	2.0	2.2		-
Fr	Ra	Ac-No																
0.7	0.9	1.1-1.7																



Smaller electronegativity



Larger electronegativity

# Ionization Process

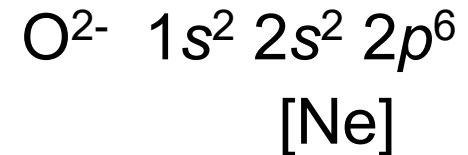
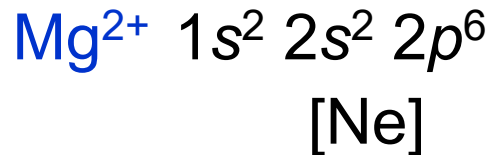
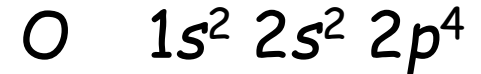
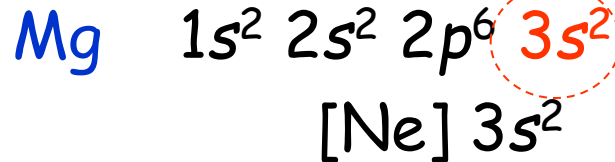
metal atom + nonmetal atom

↑  
donates  
electrons

↑  
accepts  
electrons

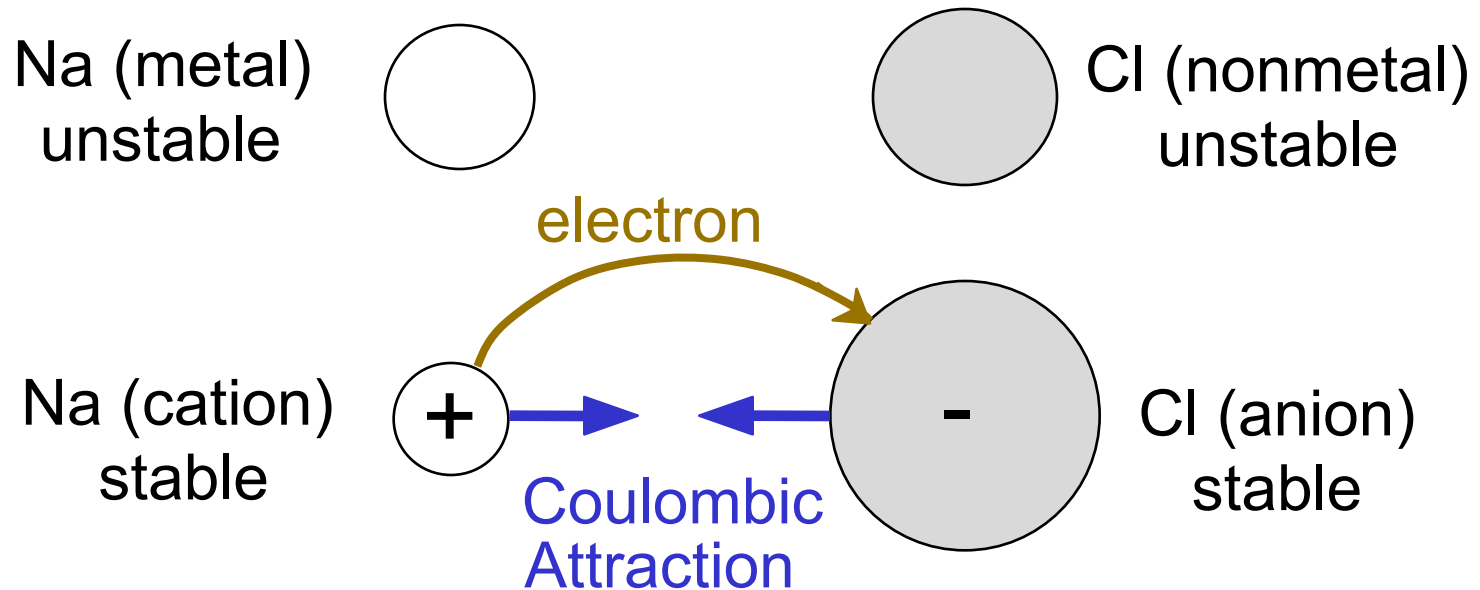
Dissimilar electronegativities

ex: MgO



# Ionic Bonding

- Occurs between + and - ions.
- Requires **electron transfer**.
- Large difference in electronegativity required.
- Example: NaCl



# Ionic Bonding (cont.)

- Energy - minimum energy most stable
  - Net energy = sum of attractive and repulsive energies
  - Equilibrium separation when net energy is a minimum

$$E_N = E_A + E_R = -\frac{A}{r} + \frac{B}{r^n}$$

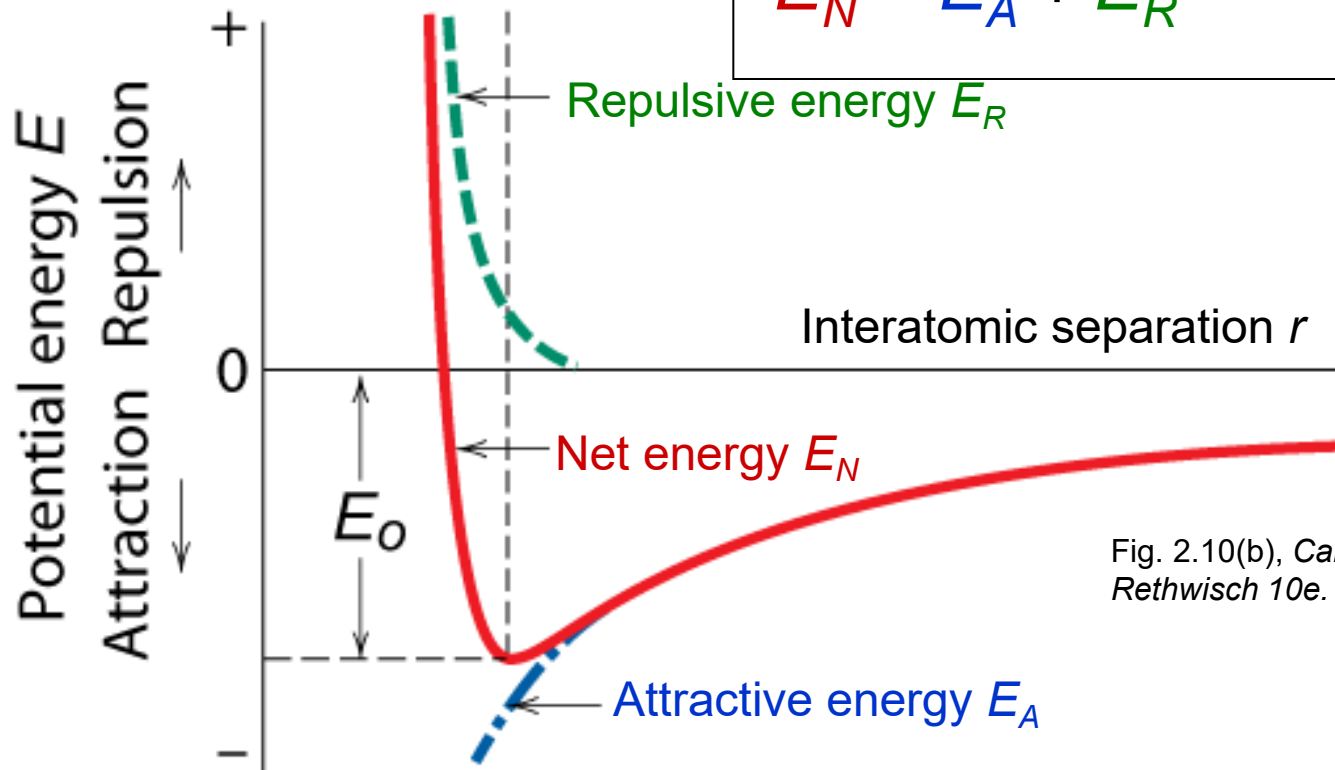


Fig. 2.10(b), Callister & Rethwisch 10e.

# Ionic Bonding (cont.)

Predominant bonding in **Ceramics**

Examples:

Periodic table showing electronegativity values (red text) and group labels (black text). Arrows indicate the bonding partners for NaCl, MgO, CaF<sub>2</sub>, and CsCl.

IA																		0
H																		He
2.1	IIA											IIIA	IVA	VA	0	VIA	VIIA	-
Li	Be											B	C	N	O	F	Ne	
1.0	1.5											2.0	2.5	3.0	3.5	4.0	-	
Na	Mg											Al	Si	P	S	Cl	Ar	
0.9	1.2	IIIB	IVB	VB	VIB	VII B	VIII			IB	IIB	1.5	1.8	2.1	2.5	3.0	-	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	
0.8	1.0	1.3	1.5	1.6	1.6	1.5	1.8	1.8	1.8	1.9	1.6	1.6	1.8	2.0	2.4	2.8	-	
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	
0.8	1.0	1.2	1.4	1.6	1.8	1.9	2.2	2.2	2.2	1.9	1.7	1.7	1.8	1.9	2.1	2.5	-	
Cs	Ba	La-Lu	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn	
0.7	0.9	1.1-1.2	1.3	1.5	1.7	1.9	2.2	2.2	2.2	2.4	1.9	1.8	1.8	1.9	2.0	2.2	-	
Fr	Ra	Ac-No																
0.7	0.9	1.1-1.7																

Give up electrons

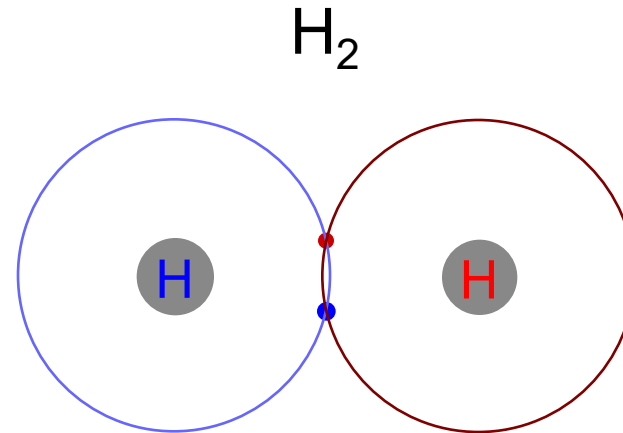
Acquire electrons

# Covalent Bonding

- Similar **electronegativities**  $\therefore$  share electrons
- Bonds involve valence electrons – normally s and p orbitals are involved
- Example:  $\text{H}_2$

Each H: has 1 valence  $e^-$ ,  
needs 1 more

Electronegativities  
are the same.

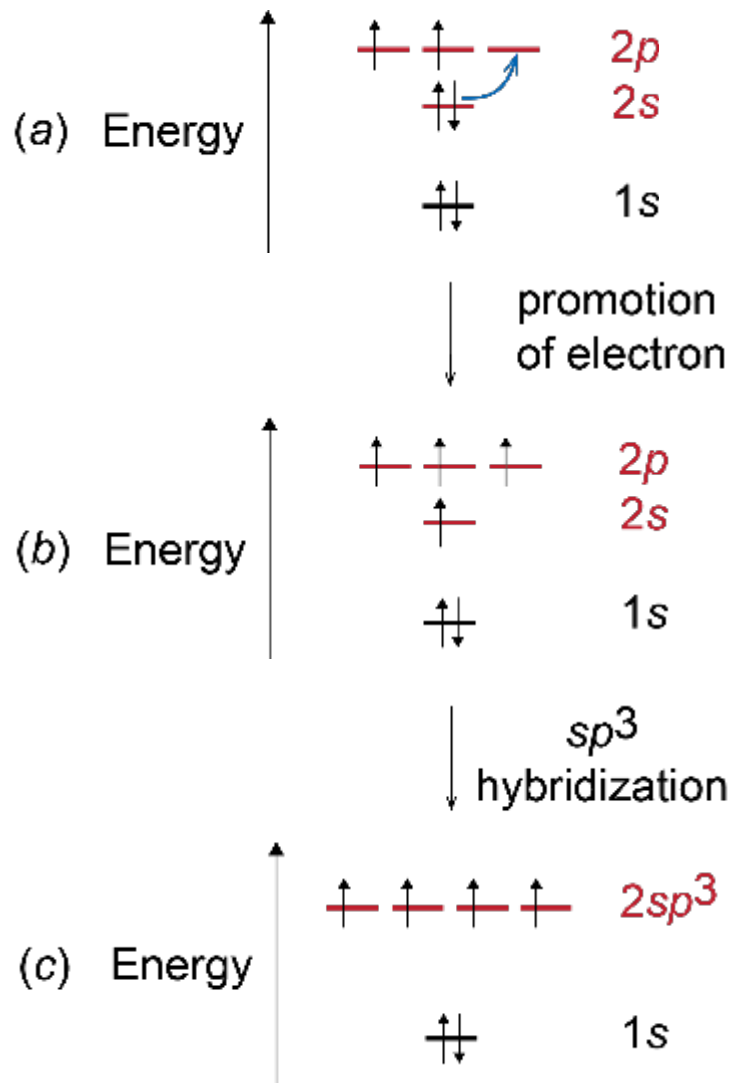


• shared 1s electron  
from 1st hydrogen  
atom

• shared 1s electron  
from 2nd hydrogen  
atom

Fig. 2.12, Calliser & Rethwisch 10e.

# Covalent Bonding: Bond Hybridization



- Carbon can form  $sp^3$  hybrid orbitals

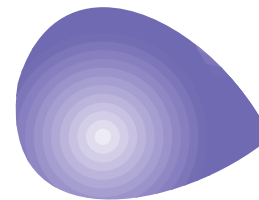


Fig. 2.14, *Callister & Rethwisch 10e*.  
(Adapted from J.E. Brady and F. Senese, *Chemistry: Matter and Its Changes*, 4<sup>th</sup> edition. Reprinted with permission of John Wiley and Sons, Inc.)

Fig. 2.13, *Callister & Rethwisch 10e*.

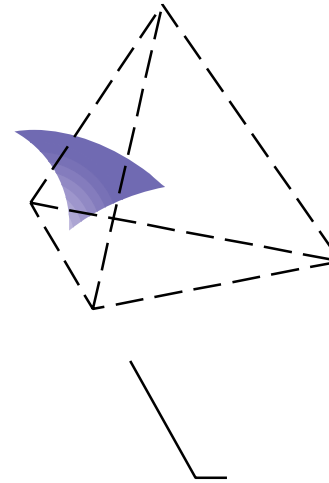
# Covalent Bonding (cont.)

## Hybrid $sp^3$ bonding involving carbon

Example:  $\text{CH}_4$

C: each has 4 valence electrons,  
needs 4 more

H: each has 1 valence electron,  
needs 1 more



Electronegativities of C and H are similar so electrons are shared in  $sp^3$  hybrid covalent bonds.

Fig. 2.15, *Callister & Rethwisch 10e*.  
(Adapted from J.E. Brady and F. Senese, *Chemistry: Matter and Its Changes*, 4<sup>th</sup> edition. Reprinted with permission of John Wiley and Sons, Inc.)

# Metallic Bonding

- **Electrons** delocalized to form an “electron cloud”

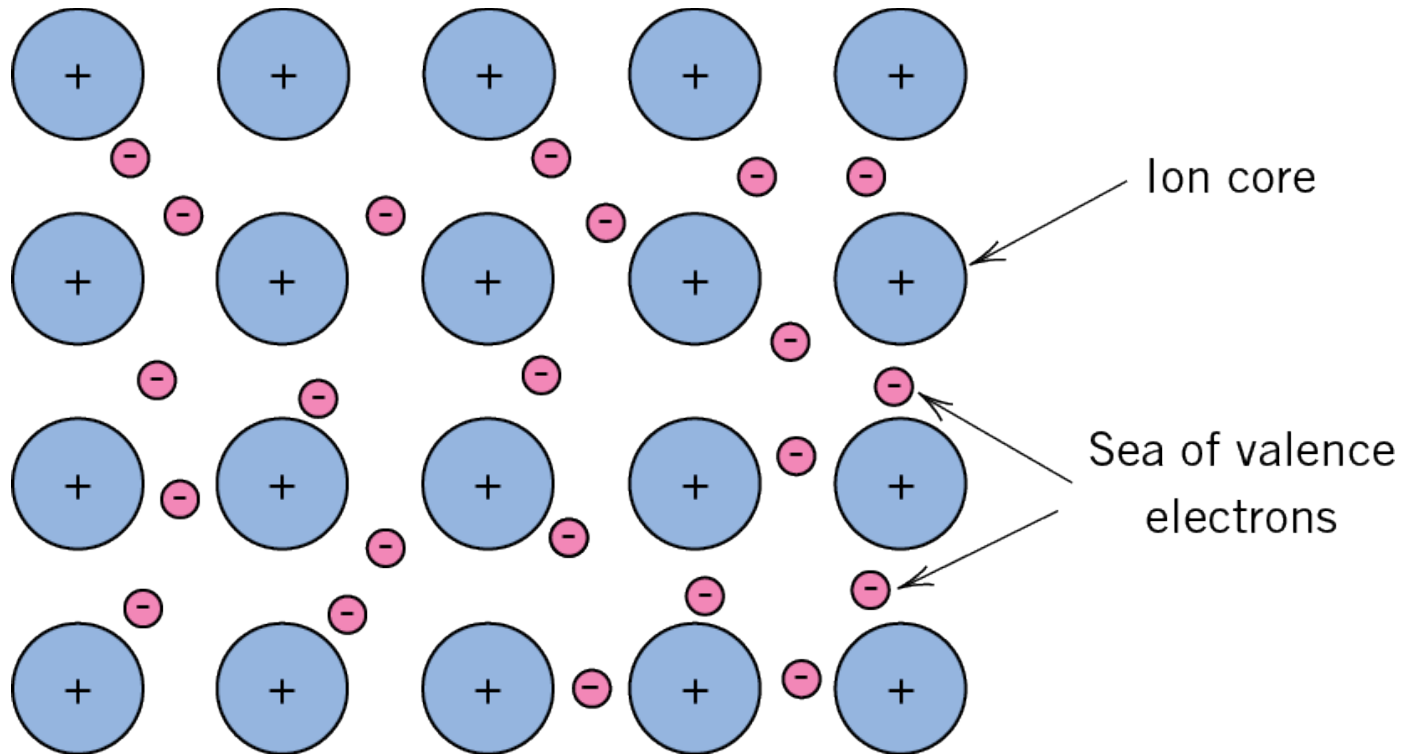


Fig. 2.19b, Callister & Rethwisch 10e.

# Mixed Bonding

- Most common mixed bonding type is Covalent-Ionic mixed bonding

$$\% \text{ ionic character} = \left( 1 - e^{-\frac{(X_A - X_B)^2}{4}} \right) \times (100\%)$$

where  $X_A$  &  $X_B$  are electronegativities of the two elements participating in the bond

Ex: MgO

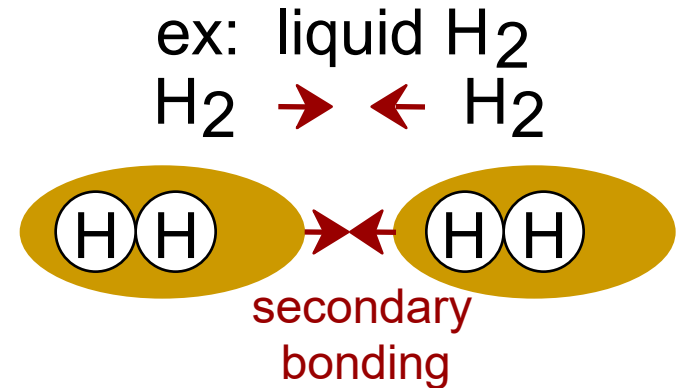
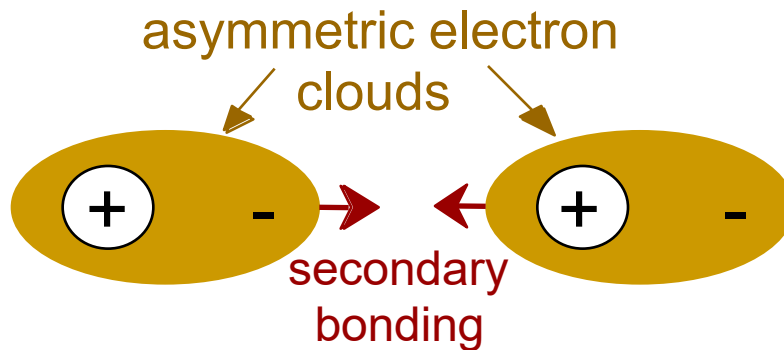
$$\begin{aligned} X_{\text{Mg}} &= 1.2 \\ X_{\text{O}} &= 3.5 \end{aligned}$$

$$\% \text{ ionic character} = \left( 1 - e^{-\frac{(3.5 - 1.2)^2}{4}} \right) \times (100\%) = 73.3\%$$

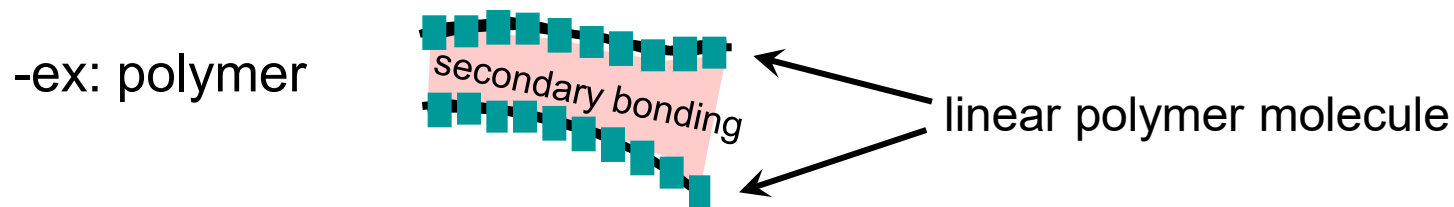
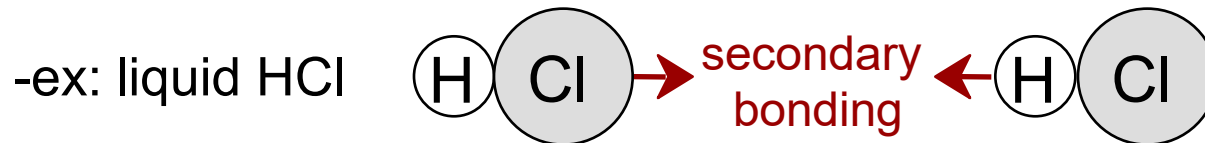
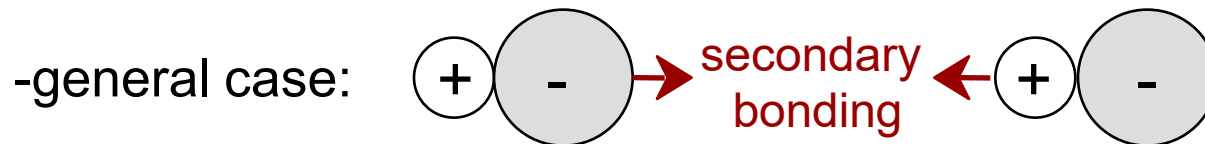
# Secondary Bonding

Arises from attractive forces between dipoles

- Fluctuating dipoles

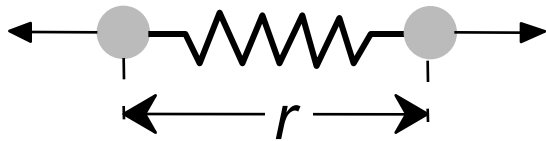


- Permanent dipoles

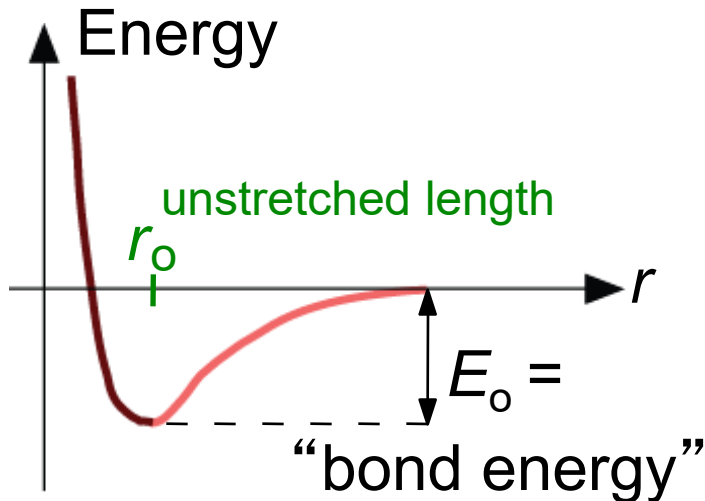


# Properties Related to Bonding I: Melting Temperature ( $T_m$ )

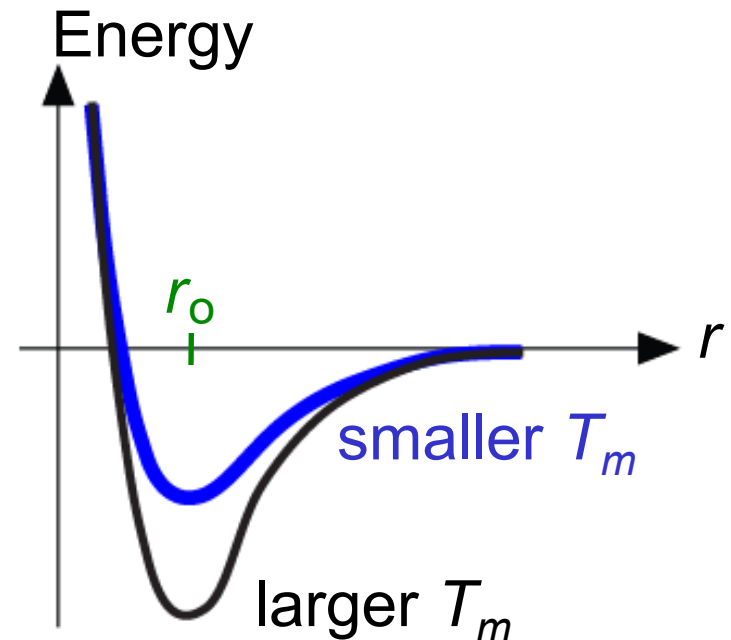
- Bond length,  $r$



- Bond energy,  $E_o$



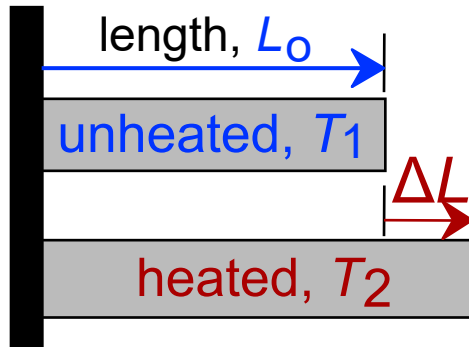
- Melting Temperature,  $T_m$



The larger  $E_o$ , the higher  $T_m$

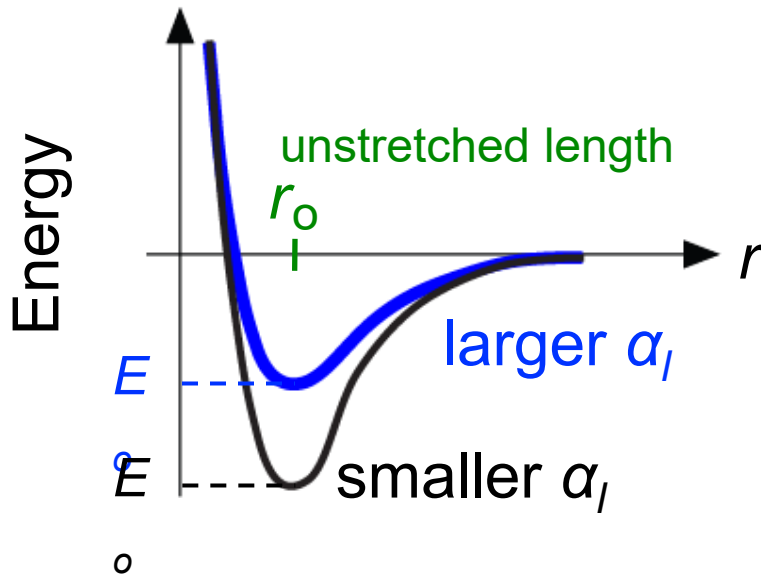
# Prop. Related to Bonding II: Coeff. of Thermal Expansion ( $\alpha_l$ )

- Coefficient of thermal expansion,  $\alpha_l$



$$\frac{\Delta L}{L_0} = \alpha_l (T_2 - T_1)$$

The smaller  $E_0$ , the larger  $\alpha_l$ .



- Increase in bond length is due to asymmetry of the  $E$  vs.  $r$  curve. This results in an increase in  $\alpha_l$ .
- As  $E_0$  increases this asymmetry decreases.

# Summary: Bonding Type and Bonding Energy

## Ceramics

(Ionic & covalent bonding):

Large bond energy

high  $T_m$

large  $E$

small  $\alpha_f$

## Metals

(Metallic bonding):

Variable bond energy

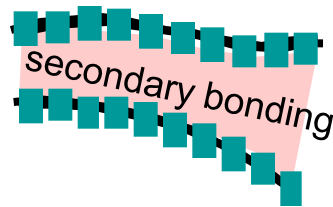
moderate  $T_m$

moderate  $E$

moderate  $\alpha_f$

## Polymers

(Covalent & Secondary):



Weak bond energy (between chains)

Secondary bonding responsible for most physical properties

low  $T_m$

small  $E$

large  $\alpha_f$

# summary

---

- A material's chemical, electrical, thermal, and optical properties are determined by electronic configuration.
- Valence electrons occupy the outermost unfilled electron shell.
- Primary bonding types include covalent, ionic, and metallic bonding.
- Secondary or van der Waals bonds are weaker than the primary bonding types.
- The percent ionic character of a covalent-ionic mixed bond between two elements depends on their electronegativities.

# The Structure of Crystalline Solids

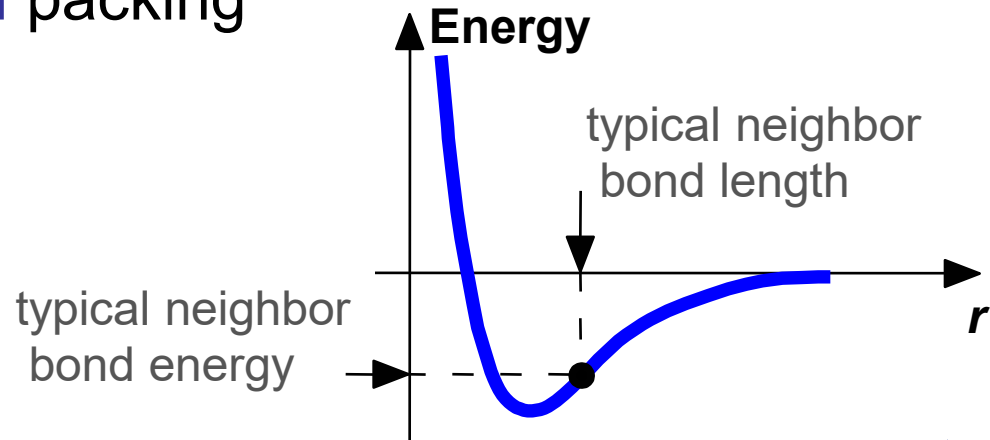
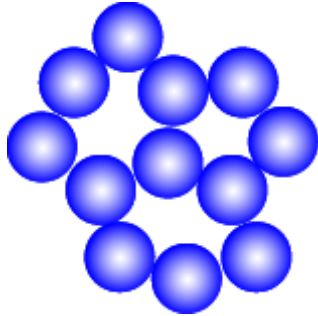
---

## ISSUES TO EXPLORE...

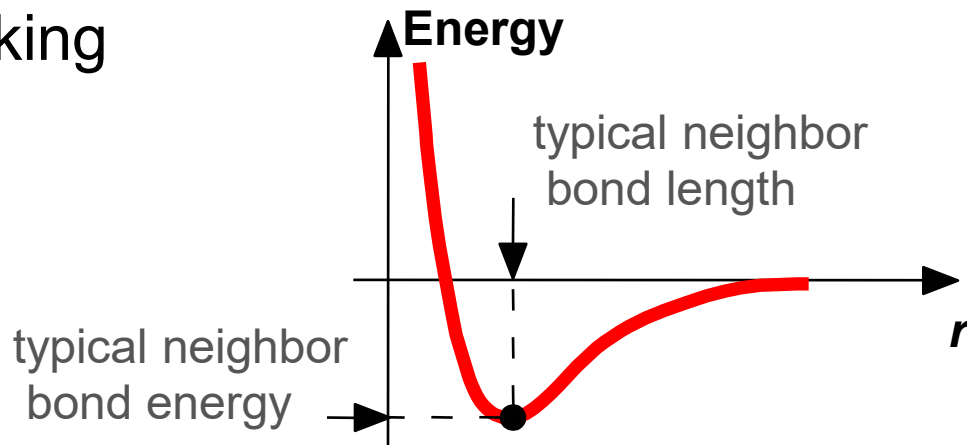
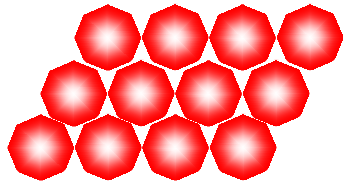
- What is the difference in atomic arrangement between crystalline and noncrystalline solids?
- What are the crystal structures of metals?
- What are the characteristics of crystal structures?
- How are crystallographic points, directions, and planes specified?
- What characteristics of a material's atomic structure determine its density?

# Energy and Packing

- Non dense, **random** packing



- Dense, **ordered** packing

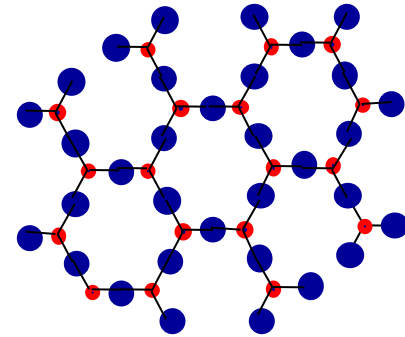


Ordered structures tend to be nearer the minimum in bonding energy and are more stable.

# Materials and Atomic Arrangements

## Crystalline materials...

- atoms arranged in periodic, 3D arrays
- typical of:
  - metals
  - many ceramics
  - some polymers



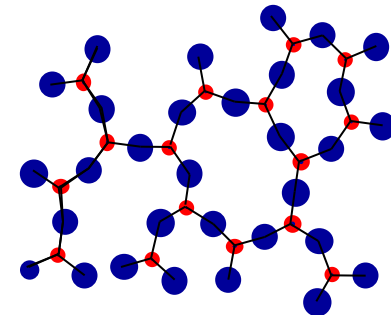
crystalline SiO<sub>2</sub>

Adapted from Fig. 3.24(a),  
*Callister & Rethwisch 10e.*

• **Si**      • **Oxygen**

## Noncrystalline materials...

- atoms have no periodic arrangement
- occurs for:
  - complex structures
  - rapid cooling



noncrystalline SiO<sub>2</sub>

Adapted from Fig. 3.24(b),  
*Callister & Rethwisch 10e.*

"Amorphous" = Noncrystalline

# Metallic Crystal Structures: Atomic Packing

- Dense atomic packing for crystal structures of metals.
- Reasons for dense packing:
  - Bonds between metal atoms are nondirectional.
  - Nearest neighbor distances tend to be small in order to lower bond energy.
  - High degree of shielding (of ion cores) provided by free electron cloud.
- Crystal structures for metals simpler than structures for ceramics and polymers.

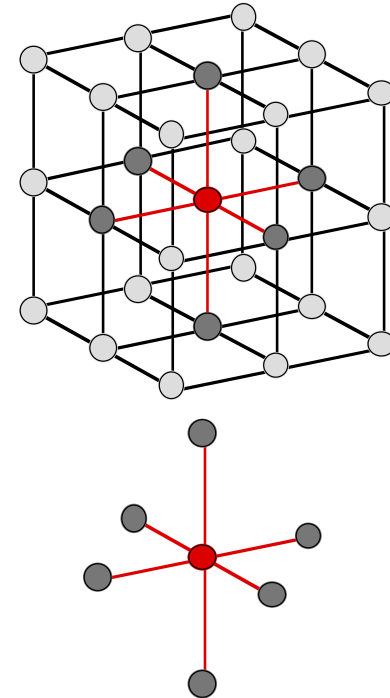
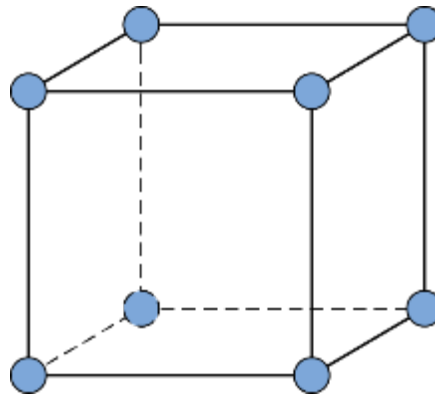
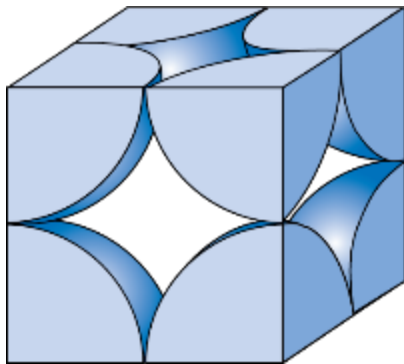
We will examine three such structures for metals...

# Simple Cubic (SC) Crystal Structure

- Centers of atoms located at the eight corners of a cube
- Rare due to low packing density (only Po has this structure)
- **Close-packed directions** are cube edges.

ex: Po

- **Coordination # = 6**  
(# nearest neighbors)



Adapted from Fig. 3.3, *Callister & Rethwisch 10e*.

# Definitions

---

## Coordination Number

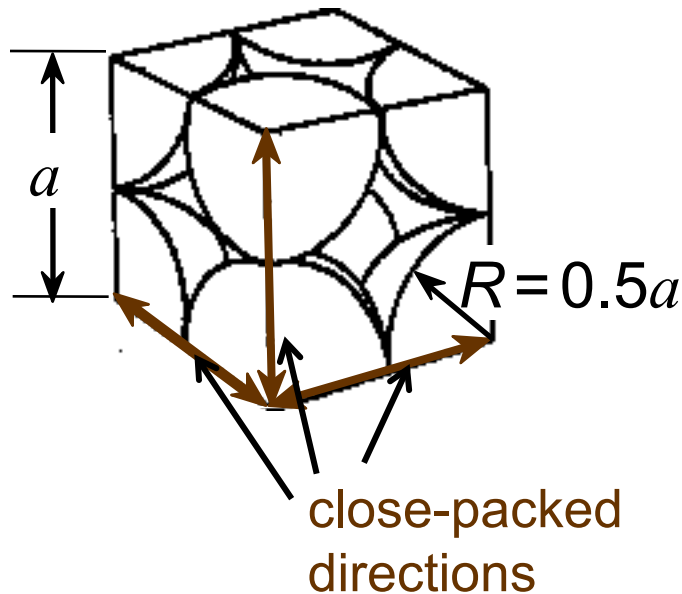
Coordination Number = number of nearest-neighbor or touching atoms

## Atomic Packing Factor (APF)

$$\text{APF} = \frac{\text{Volume of atoms in unit cell}^*}{\text{Volume of unit cell}}$$

\*assume hard spheres

# Atomic Packing Factor (APF) for Simple Cubic



$$\text{APF} = \frac{\text{atoms unit cell} \times \frac{4}{3} \pi (0.5a)^3}{a^3} = 0.52$$

The diagram shows the calculation of the Atomic Packing Factor (APF) for a simple cubic unit cell. The numerator represents the total volume of atoms in the unit cell, calculated as the number of atoms per unit cell (1) multiplied by the volume of a single atom ( $\frac{4}{3} \pi (0.5a)^3$ ). The denominator represents the volume of the unit cell ( $a^3$ ). The final result is 0.52.

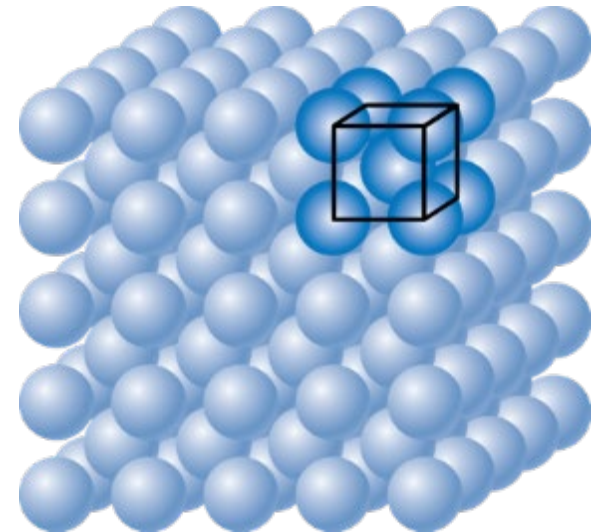
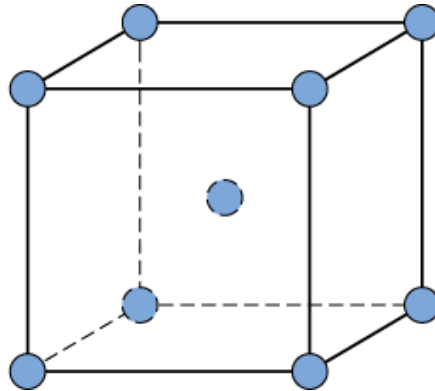
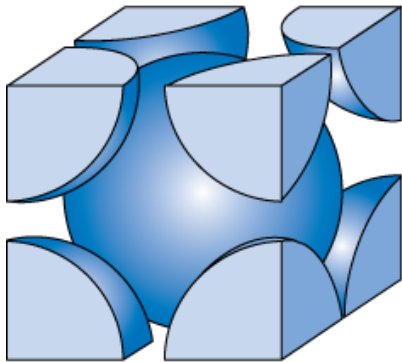
Unit cell contains 1 atom =  $8 \times 1/8 = 1$  atom/unit cell

# Body-Centered Cubic Structure (BCC)

- Atoms located at 8 cube corners with a single atom at cube center.  
--Note: All atoms in the animation are identical; the center atom is shaded differently for ease of viewing.

ex: Cr, W, Fe ( $\alpha$ ), Ta, Mo

- Coordination # = 8

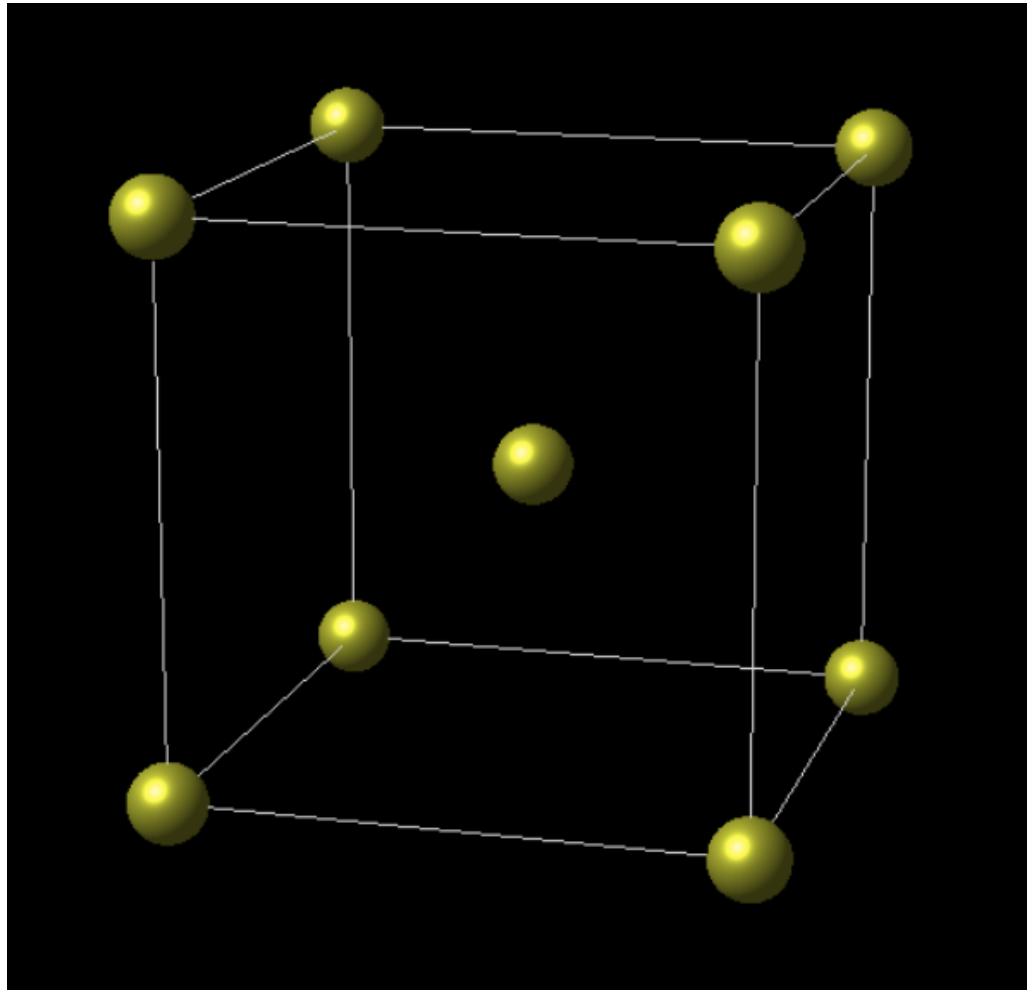


Adapted from Fig. 3.2, *Callister & Rethwisch 10e*.

2 atoms/unit cell: 1 center + 8 corners  $\times$  1/8

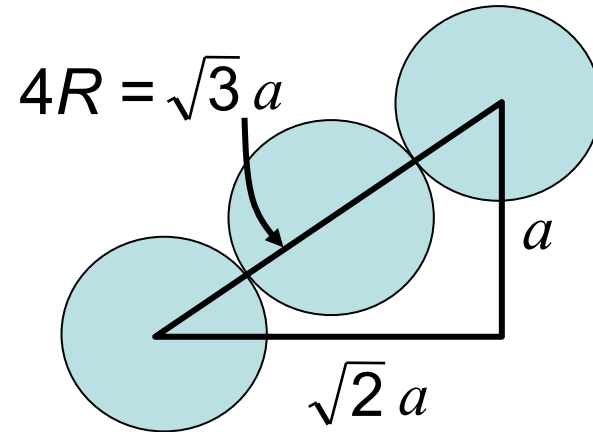
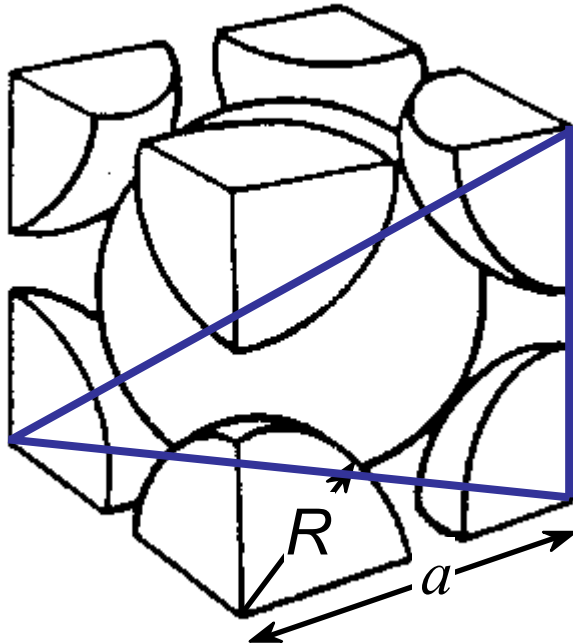
# VMSE Screenshot - BCC Unit Cell

---



# Atomic Packing Factor: BCC

- APF for the body-centered cubic structure = 0.68



For close-packed directions

$$R = \sqrt{3} a/4$$

atoms  
unit cell

$$\text{APF} = \frac{2 \cdot \frac{4}{3} \pi (\sqrt{3}a/4)^3}{a^3}$$

volume  
unit cell

$$a^3$$

volume

atom

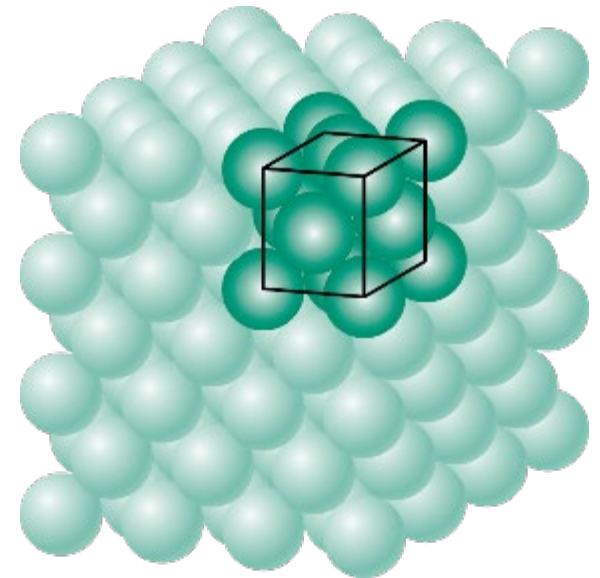
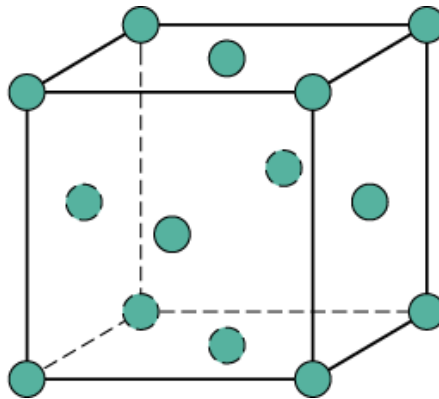
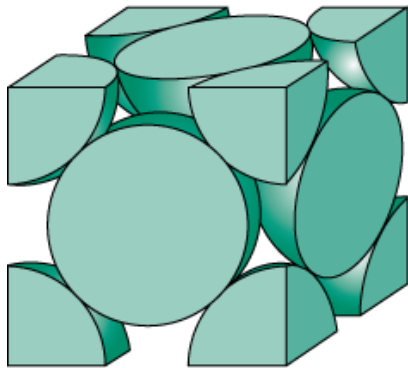
$$= \frac{8}{3} \pi \left( \frac{\sqrt{3}}{4} \right)^3 = 0.68$$

# Face-Centered Cubic Structure (FCC)

- Atoms located at 8 cube corners and at the centers of the 6 faces.  
--Note: All atoms in the animation are identical; the face-centered atoms are shaded differently for ease of viewing.

ex: Al, Cu, Au, Pb, Ni, Pt, Ag

- Coordination # = 12

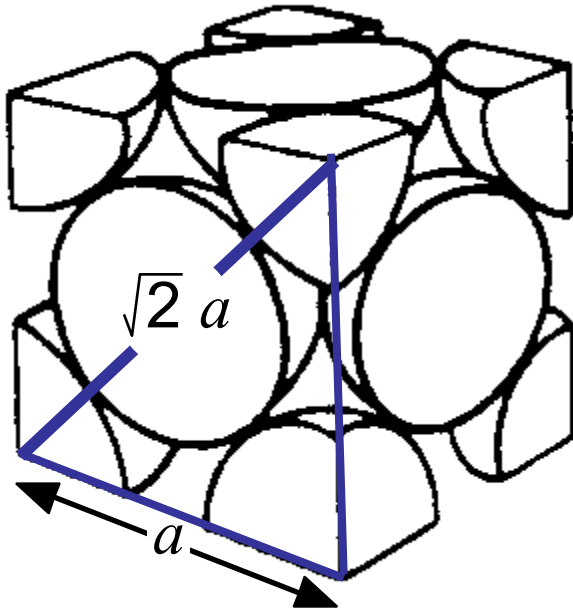


Adapted from Fig. 3.1, *Callister & Rethwisch 10e*.

4 atoms/unit cell:  $6 \text{ face} \times 1/2 + 8 \text{ corners} \times 1/8$

# Atomic Packing Factor: FCC

- APF for the face-centered cubic structure = 0.74  
maximum achievable APF



For close-packed directions:

$$4R = \sqrt{2} a \quad \left( \text{i.e., } R = \frac{\sqrt{2}a}{4} \right)$$

Unit cell contains:  $6 \times 1/2 + 8 \times 1/8$   
= 4 atoms/unit cell

$$\text{APF} = \frac{\text{atoms/unit cell} \times \text{volume/atom}}{\text{volume/unit cell}} = \frac{4 \times \frac{4}{3} \pi \left(\frac{\sqrt{2}a}{4}\right)^3}{a^3} = 0.74$$

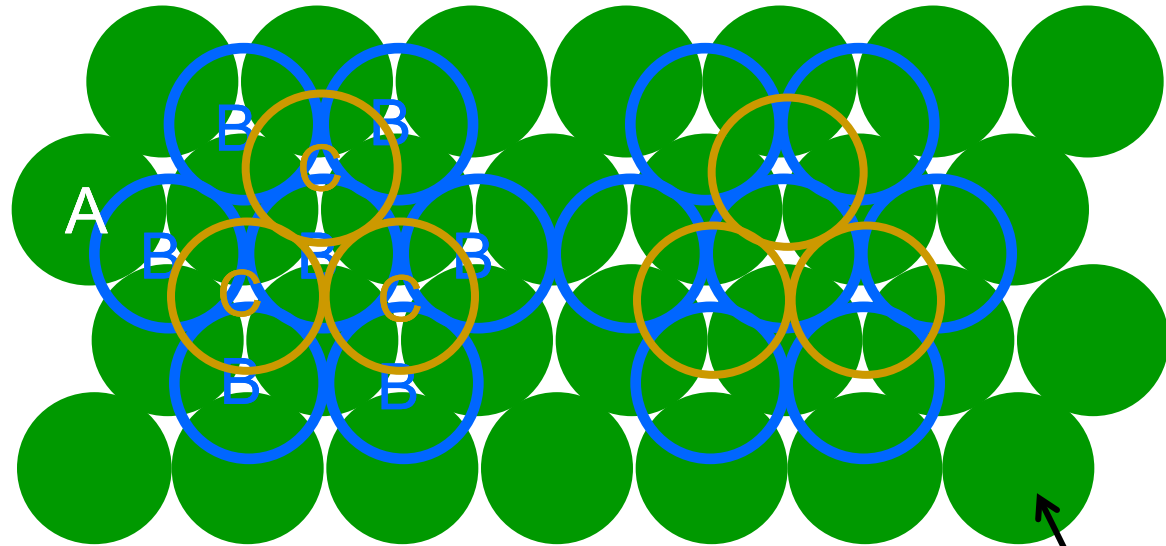
# FCC Plane Stacking Sequence

- ABCABC... Stacking Sequence—Close-Packed Planes of Atoms
- 2D Projection

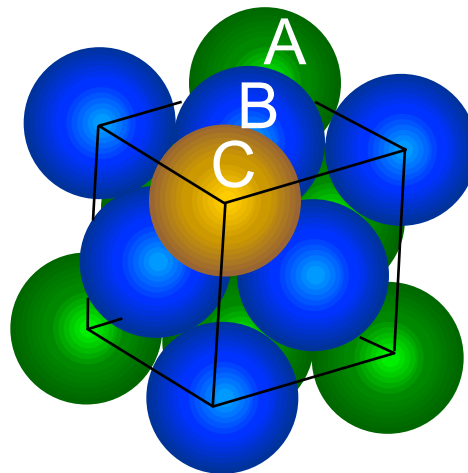
A sites

B sites

C sites



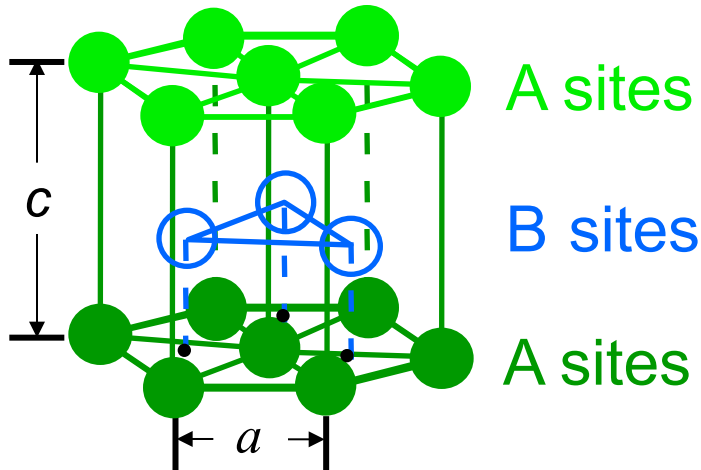
- Stacking Sequence Referenced to an FCC Unit Cell.



Close-Packed Plane

# Hexagonal Close-Packed Structure (HCP)

- ABAB... Stacking Sequence—Close-Packed Planes of Atoms
- 3D Projection
- 2D Projection



- Coordination # = 12
- APF = 0.74
- Ideal  $c/a = 1.633$

6 atoms/unit cell

ex: Cd, Mg, Ti, Zn

# Theoretical Density for Metals, $\rho$

$$\text{Density} = \rho = \frac{\text{Mass of Atoms in Unit Cell}}{\text{Total Volume of Unit Cell}} = \frac{(nA / N_A)}{V_C}$$

$$\rho = \frac{nA}{V_C N_A}$$

where

$n$  = number of atoms/unit cell

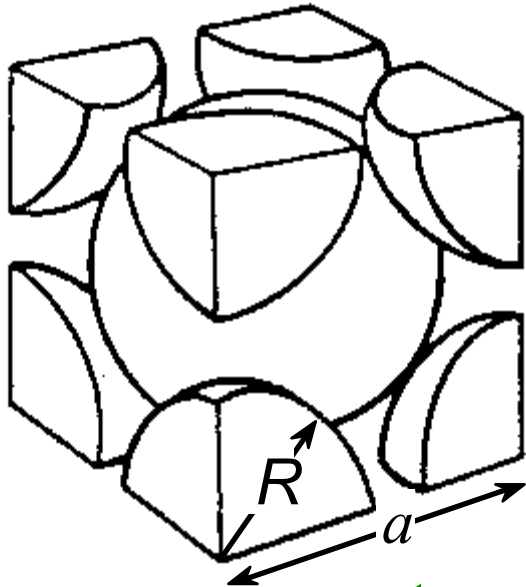
$A$  = atomic weight

$V_C$  = Volume of unit cell =  $a^3$  for cubic

$N_A$  = Avogadro's number

=  $6.022 \times 10^{23}$  atoms/mol

# Theoretical Density Computation for Chromium



- Cr has BCC crystal structure

$$A = 52.00 \text{ g/mol}$$

$$R = 0.125 \text{ nm}$$

$$n = 2 \text{ atoms/unit cell}$$

$$a = 4R/\sqrt{3} = 0.2887 \text{ nm}$$

$$V_C = a^3 = 2.406 \times 10^{-23} \text{ cm}^3$$

$$\rho = \frac{\overbrace{\begin{matrix} n & A \\ \hline \end{matrix}}^{\substack{\text{atoms} \\ \text{unit cell}}} = \frac{\begin{matrix} 2 & 52.00 \end{matrix}}{\underbrace{\begin{matrix} 2.406 \times 10^{-23} & 6.022 \times 10^{23} \\ \hline \end{matrix}}_{\substack{\text{volume} \\ \text{unit cell}}}} = 7.19 \text{ g/cm}^3$$

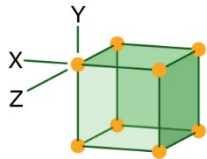
$\frac{\text{g}}{\text{mol}}$

$\frac{\text{atoms}}{\text{mol}}$

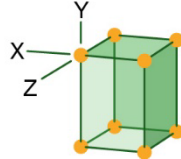
$\rho_{\text{actual}} = 7.15 \text{ g/cm}^3$

# Crystalline systems

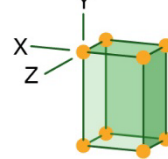
In crystallography, we distinguish 7 crystalline systems that correspond to groups of symmetry:



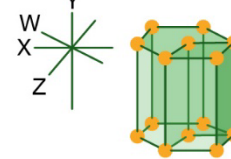
**Isometric (or cubic)**  
All three axes are equal in length, and all are perpendicular to one another.



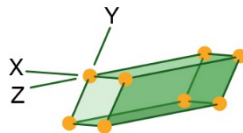
**Tetragonal**  
Two of the three axes are equal in length, and all three axes are perpendicular to one another.



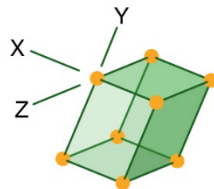
**Orthorhombic**  
All three axes are unequal in length, and all are perpendicular to one another.



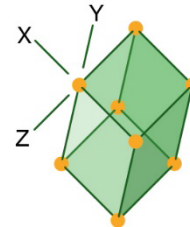
**Hexagonal**  
Of four axes, three are of equal length, are separated by equal angles, and lie in the same plane. The fourth axis is perpendicular to the plane of the other three axes. Hexagonal cells have lattice points in each of the two six-sided faces.



**Triclinic**  
All three axes are unequal in length, and none is perpendicular to another.



**Monoclinic**  
All three axes are unequal in length, and two axes are perpendicular to each other.



**Rhombohedral (or trigonal)\***  
All three axes are of equal length, and none of the axes is perpendicular to another, but the crystal faces all have the same size and shape.

© Encyclopædia Britannica, Inc.

\*Some sources do not separate the hexagonal and rhombohedral (trigonal) systems.

# Densities Comparison for Four Material Types

In general

$$\rho_{\text{metals}} > \rho_{\text{ceramics}} > \rho_{\text{polymers}}$$

Why?

Metals have...

- close-packing (metallic bonding)
- often large atomic masses

Ceramics have...

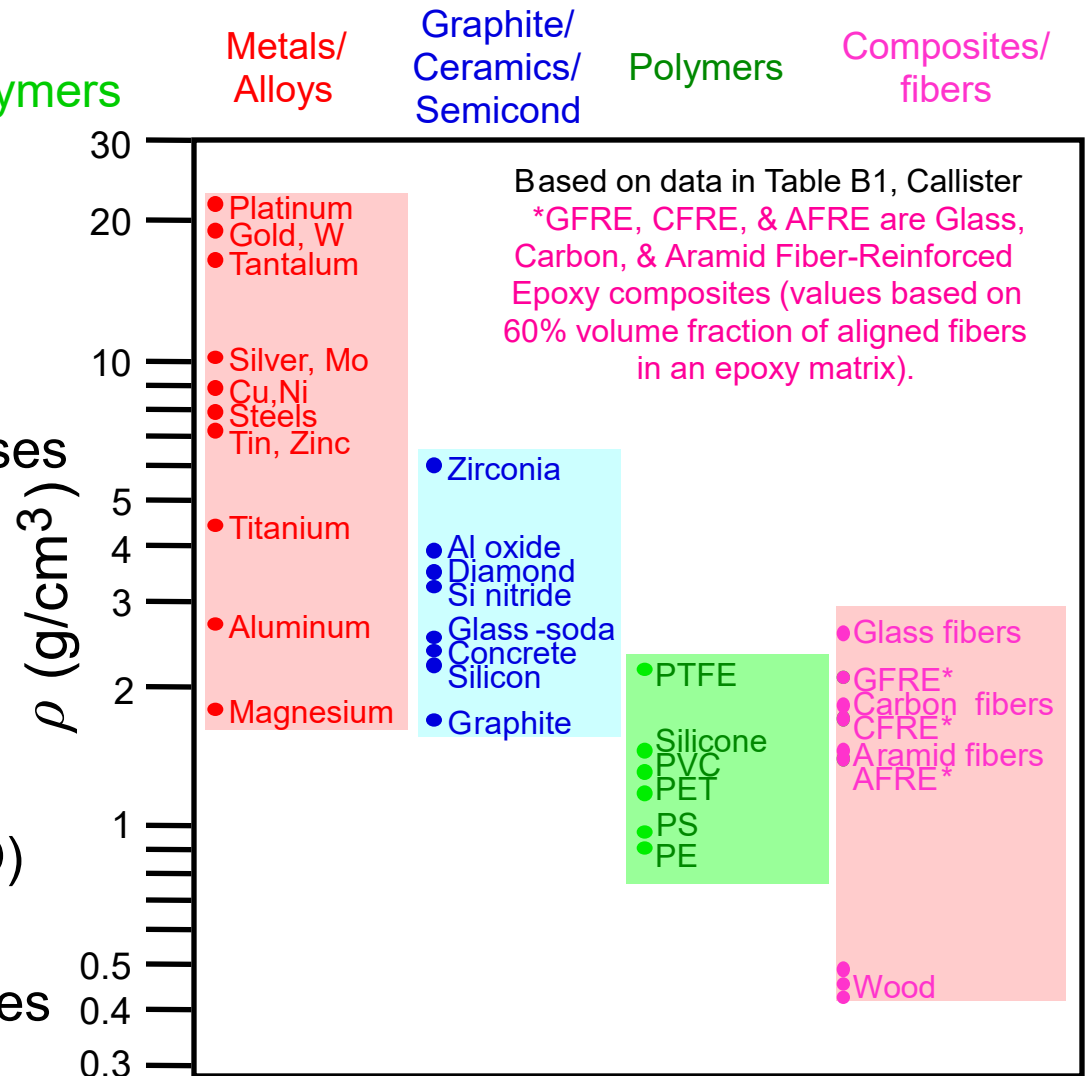
- often lighter elements

Polymers have...

- low packing density (often amorphous)
- lighter elements (C,H,O)

Composites have...

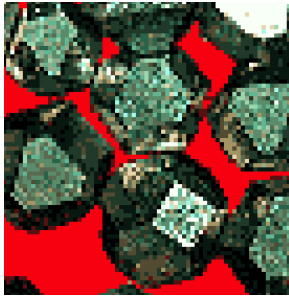
- moderate to low densities



# Single Crystals

- When the periodic arrangement of atoms (crystal structure) extends without interruption throughout the entire specimen.

-- diamond single crystals for abrasives



(Courtesy Martin Deakins, GE Superabrasives, Worthington, OH. Used with permission.)

-- Quartz single crystal



(Courtesy P.M. Anderson)

-- single crystal for turbine blade



Fig. 8.35(c), *Callister & Rethwisch 10e.*  
(courtesy of Pratt and Whitney)

# Polycrystalline Materials

- *Most* engineering materials are composed of many small, single crystals (i.e., are *polycrystalline*).

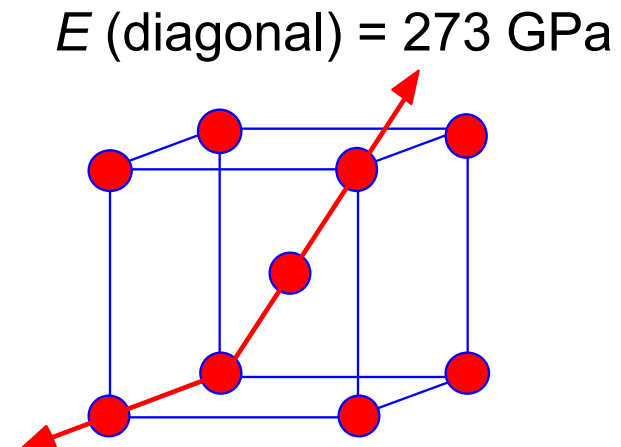


- Nb-Hf-W plate with an electron beam weld.
- Each "grain" is a single crystal.
- Grain sizes typically range from 1 nm to 2 cm (i.e., from a few to millions of atomic layers).

# Anisotropy

- Anisotropy — Property value depends on crystallographic direction of measurement.
  - Observed in single crystals.
  - Example: modulus of elasticity ( $E$ ) in BCC iron

$$E(\text{edge}) \neq E(\text{diagonal})$$



$E(\text{edge}) = 125 \text{ GPa}$

Unit cell of BCC iron

# Isotropy

- Polycrystals
  - Properties may/may not vary with direction.
  - If grains randomly oriented: properties **isotropic**.  
( $E_{\text{poly iron}} = 210 \text{ GPa}$ )
  - If grains **textured** (e.g., deformed grains have preferential crystallographic orientation): properties **anisotropic**.

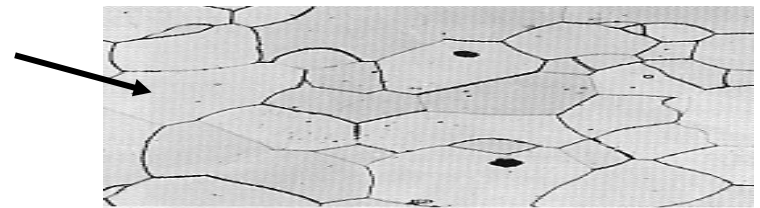
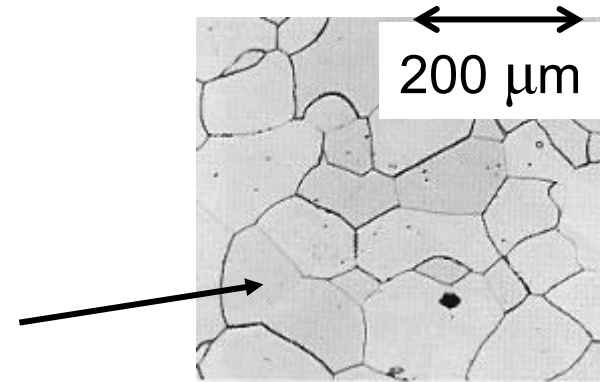


Fig. 4.15(b), *Callister & Rethwisch 10e*.  
[Fig. 4.15(b) is courtesy of L.C. Smith and C. Brady, the National Bureau of Standards, Washington, DC (now the National Institute of Standards and Technology, Gaithersburg, MD).]

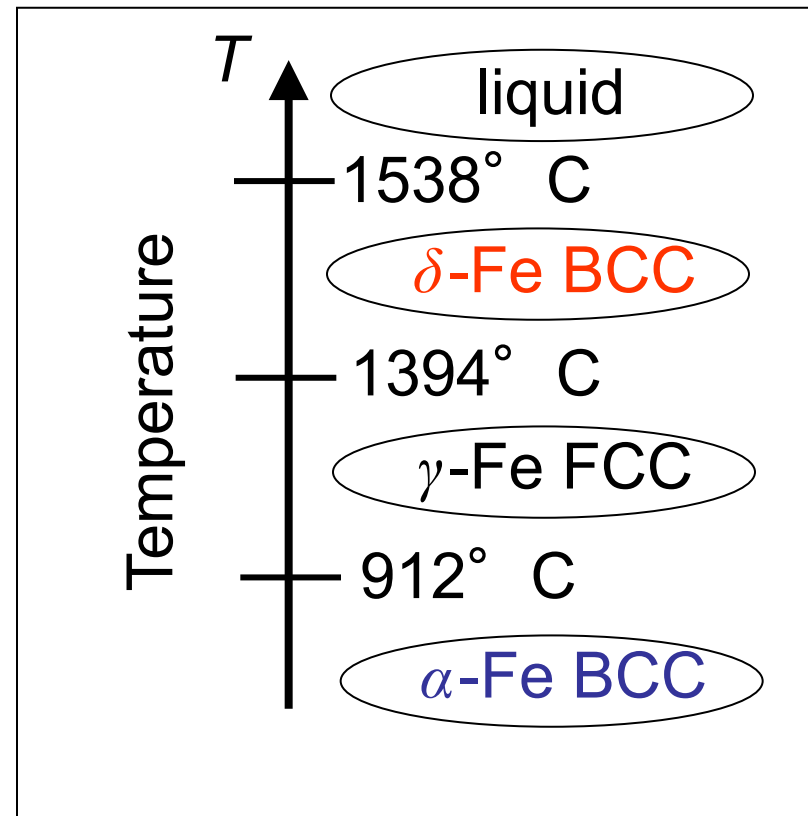
# Polymorphism/Allotropy

- Two or more distinct crystal structures for the same material (allotropy/polymorphism)

Iron system

Titanium:  $\alpha$  or  $\beta$  forms

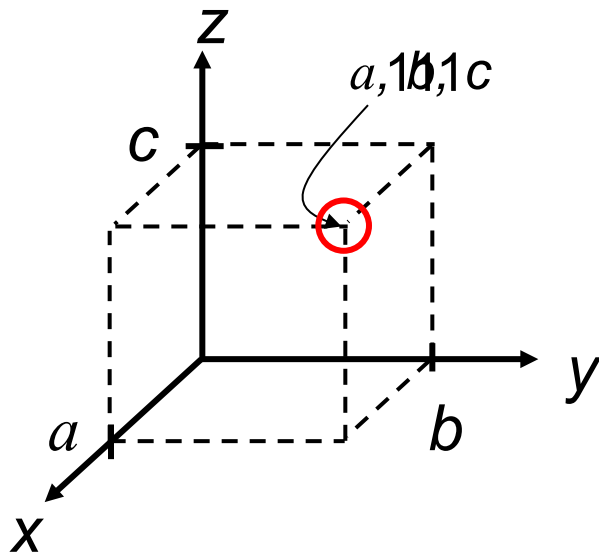
Carbon:  
diamond, graphite



# Point Coordinates

A **point coordinate** is a lattice position in a unit cell

Determined as fractional multiples of  $a$ ,  $b$ , and  $c$  unit cell edge lengths



Example: Unit cell upper corner

1. Lattice position is

$$a, b, c$$

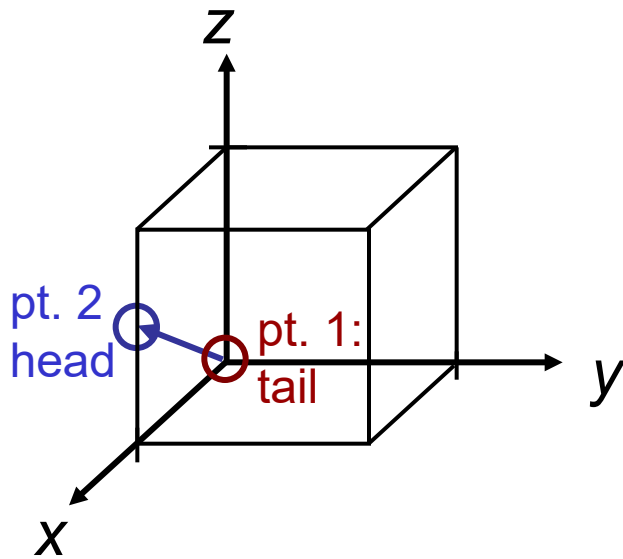
2. Divide by unit cell edge lengths ( $a$ ,  $b$ , and  $c$ ) and remove commas

$$\frac{a}{a} \frac{b}{b} \frac{c}{c} = 111$$

3. Point coordinates for unit cell corner are **111**

# Crystallographic Directions I.

## Example Problem I



ex:

pt. 1  $x_1 = 0, y_1 = 0, z_1 = 0$

pt. 2  $x_2 = a, y_2 = 0, z_2 = c/2$

$$\frac{a-0}{a} \quad \frac{0-0}{b} \quad \frac{c/2-0}{c}$$

## Algorithm – determine direction indices

1. Determine coordinates of vector tail, pt. 1:  $x_1, y_1, & z_1$ ; and vector head, pt. 2:  $x_2, y_2, & z_2$ .
2. Tail point coordinates subtracted from head point coordinates.
3. Normalize coordinate differences in terms of lattice parameters  $a, b,$  and  $c$ :

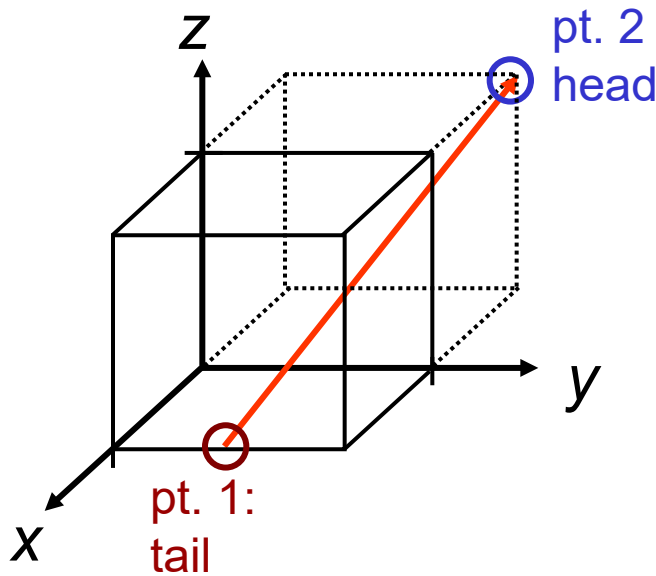
$$\frac{x_2 - x_1}{a} \quad \frac{y_2 - y_1}{b} \quad \frac{z_2 - z_1}{c}$$

4. Reduce to smallest integer values
  5. Enclose indices in square brackets, no commas
- $[uvw]$

$$\Rightarrow 1, 0, 1/2 \quad \Rightarrow 2, 0, 1$$

$$\Rightarrow [201]$$

# Crystallographic Directions II: Example Problem 2



1. Point coordinates of tail and head
- |      |       |              |               |           |
|------|-------|--------------|---------------|-----------|
| tail | pt. 1 | $x_1 = a$ ,  | $y_1 = b/2$ , | $z_1 = 0$ |
| head | pt. 2 | $x_2 = -a$ , | $y_2 = b$ ,   | $z_2 = c$ |

2 & 3. Subtract and normalize

$$\frac{-a-a}{a} = -2; \quad \frac{b-b/2}{b} = 1/2; \quad \frac{c-0}{c} = 1$$

$$\Rightarrow -2, 1/2, 1$$

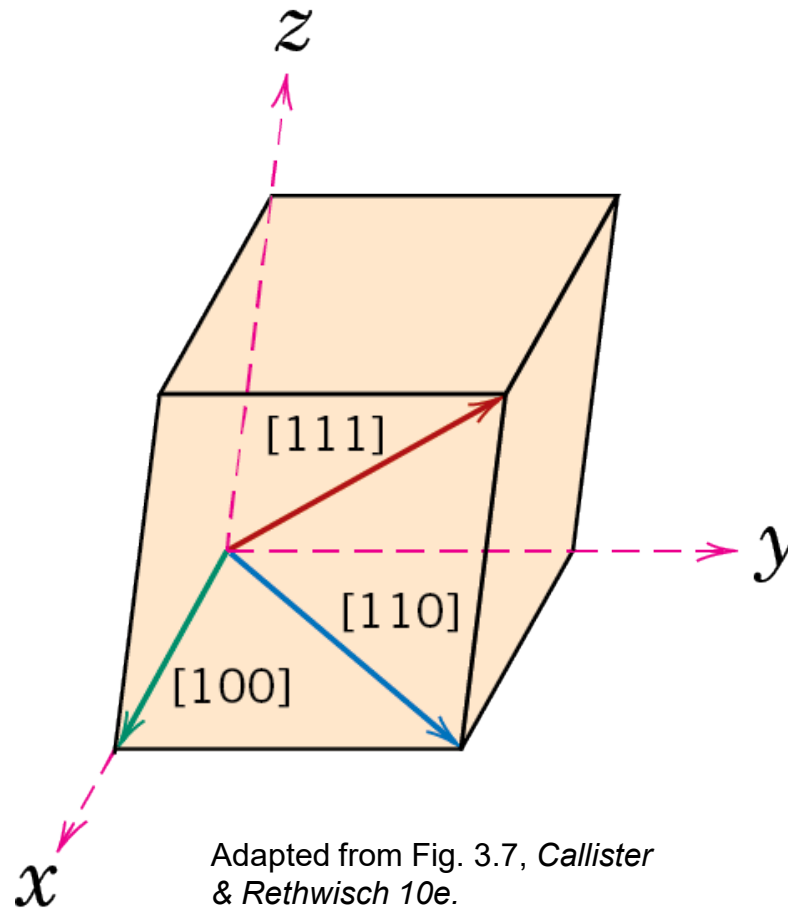
4 & 5. Multiply by 2 to eliminate the fraction, then place in square brackets (no commas)

$$-4, 1, 2 \Rightarrow [\bar{4}12] \quad \text{where the overbar represents a negative index}$$

**Family of directions** – all directions that are crystallographically equivalent (have the same atomic spacing) – indicated by indices in angle brackets

$$\text{Ex: } \langle 100 \rangle = [100], [010], [001], [\bar{1}00], [0\bar{1}0], [00\bar{1}]$$

# Common Crystallographic Directions

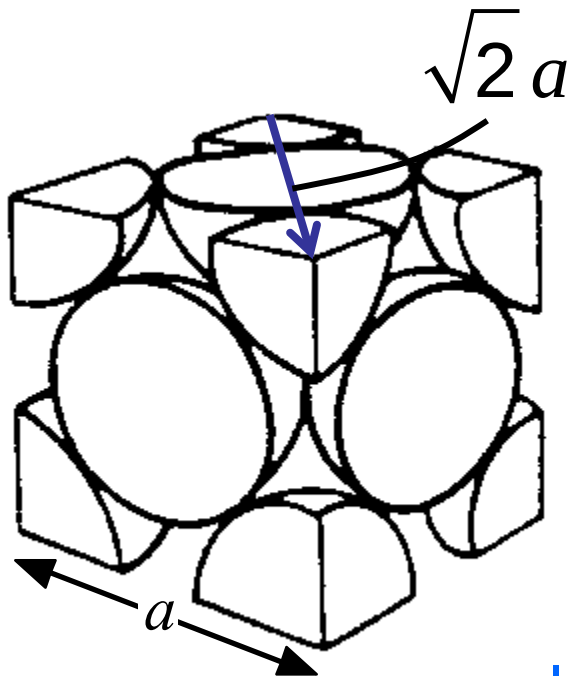


# Linear Density of Atoms (LD)

$$\text{LD} = \frac{\text{number of atoms centered on direction vector}}{\text{length of direction vector}}$$

ex: linear density of Al in [110] direction

There are 2 half atoms and 1 full atom  
= 2 atoms centered on vector



$$a = 0.405 \text{ nm}$$

$$\text{LD} = \frac{\text{\# atoms}}{\text{length}} = \frac{2}{\sqrt{2}a} = \frac{2}{\sqrt{2}(0.405 \text{ nm})} = 3.5 \text{ nm}^{-1}$$

# Crystallographic Planes

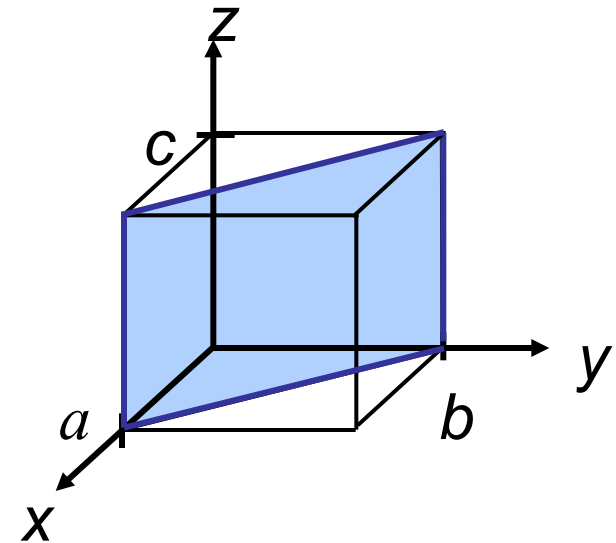
## Algorithm for determining the Miller Indices of a plane

1. If plane passes through selected origin, establish a new origin in another unit cell
2. Read off values of intercepts of plane (designated  $A, B, C$ ) with  $x, y,$  and  $z$  axes in terms of  $a, b, c$
3. Take reciprocals of intercepts
4. Normalize reciprocals of intercepts by multiplying by lattice parameters  $a, b,$  and  $c$
5. Reduce to smallest integer values
6. Enclose resulting Miller Indices in parentheses, no commas i.e.,  $(hkl)$

# Crystallographic Planes

## Example Problem I

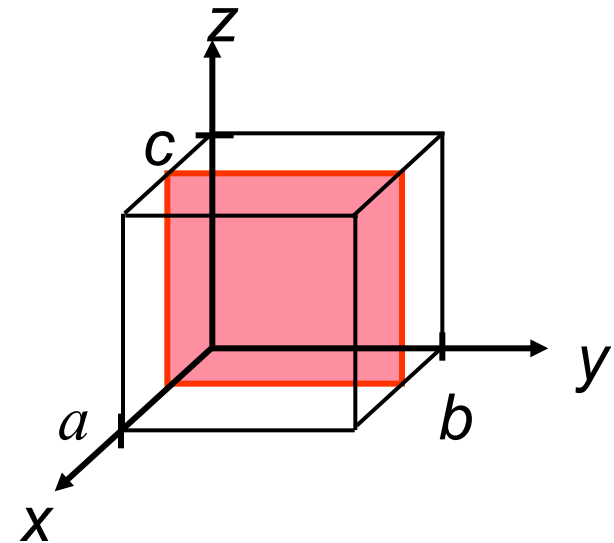
	<i>x</i>	<i>y</i>	<i>z</i>
1. Relocate origin – not needed			
2. Intercepts	<i>a</i>	<i>b</i>	$\infty c$
3. Reciprocals	$1/a$	$1/b$	$1/\infty c$
4. Normalize	$a/a$	$b/b$	$c/\infty c$
	1	1	0
5. Reduction	1	1	0
6. Miller Indices	(110)		



# Crystallographic Planes

## Example Problem II

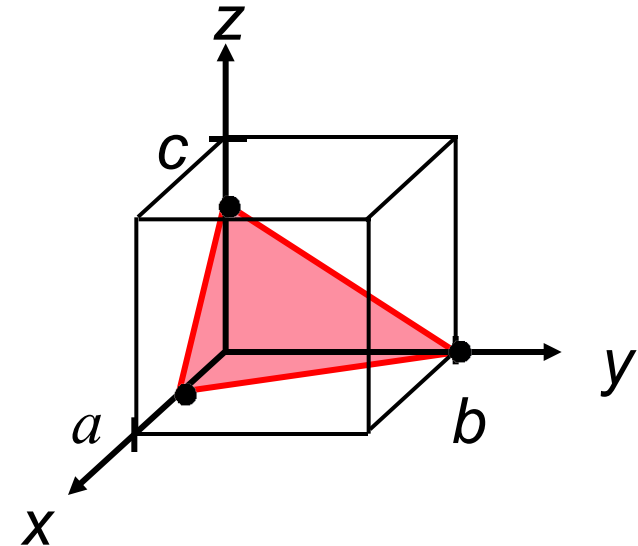
	<i>x</i>	<i>y</i>	<i>z</i>
1. Relocate origin – not needed			
2. Intercepts	$a/2$	$\infty b$	$\infty c$
3. Reciprocals	$2/a$	$1/\infty b$	$1/\infty c$
4. Normalize	$2a/a$	$b/\infty b$	$c/\infty c$
	2	0	0
5. Reduction	2	0	0
6. Miller Indices	(200)		



# Crystallographic Planes

## Example Problem III

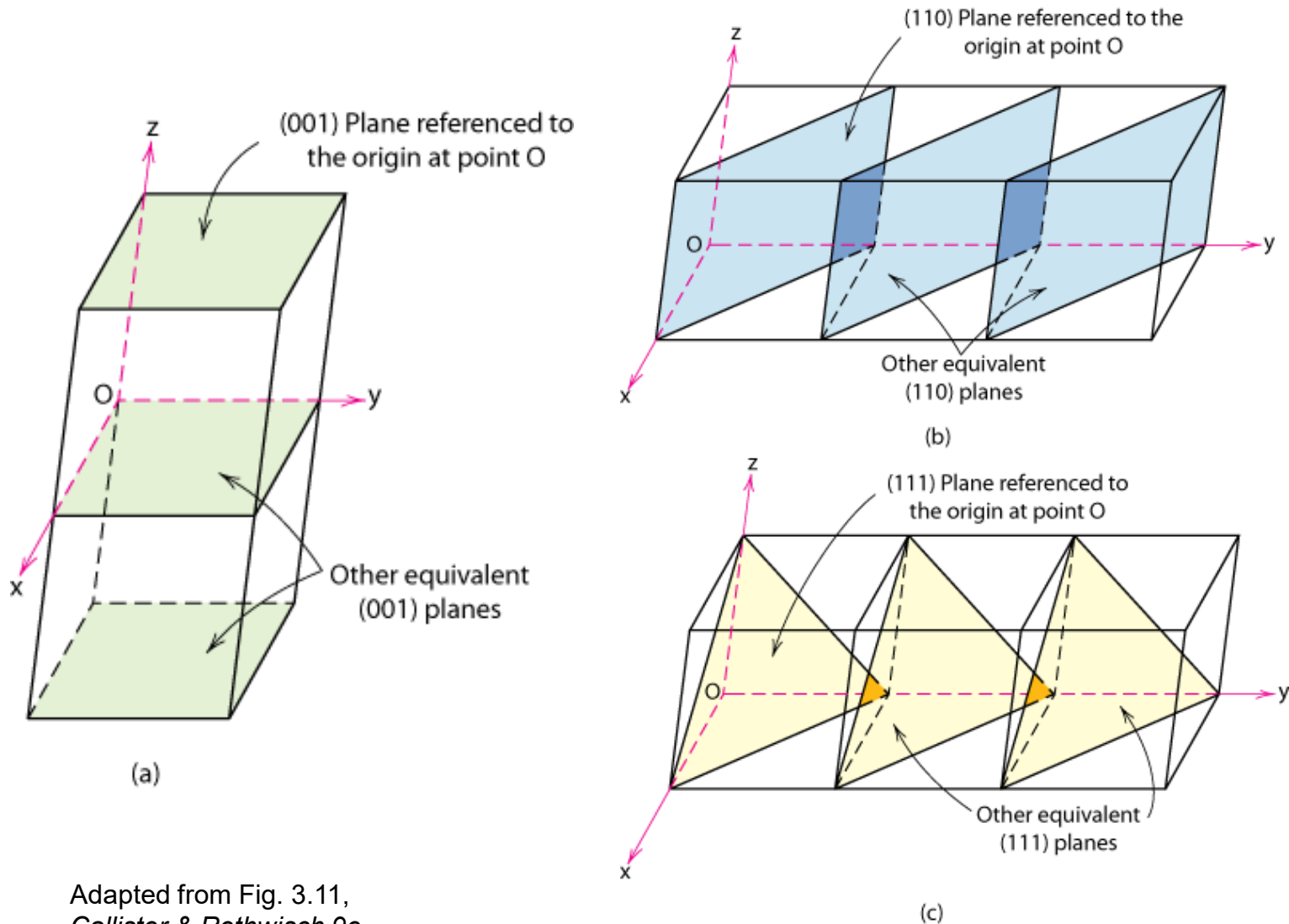
	<i>x</i>	<i>y</i>	<i>z</i>
1. Relocate origin – not needed			
2. Intercepts	$a/2$	$b$	$3c/4$
3. Reciprocals	$2/a$	$1/b$	$4/3c$
4. Normalize	$2a/a$	$b/b$	$4c/3c$
	2	1	4/3
5. Reduction (x3)	6	3	4
6. Miller Indices	(634)		



**Family of planes** – all planes that are crystallographically equivalent (have the same atomic packing) – indicated by indices in braces

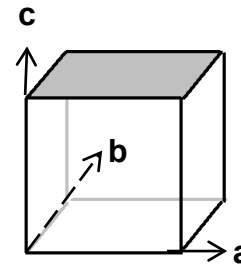
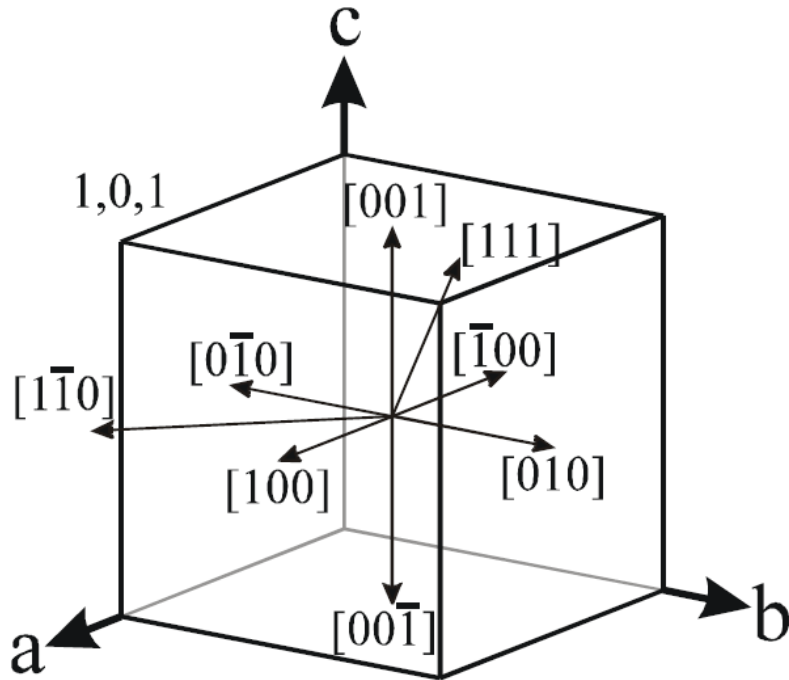
Ex:  $\{100\} = (100), (010), (001), (\bar{1}00), (0\bar{1}0), (00\bar{1})$

# Common Crystallographic Planes

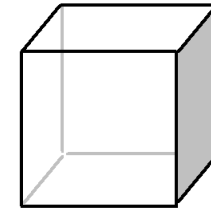


Adapted from Fig. 3.11,  
*Callister & Rethwisch 9e.*

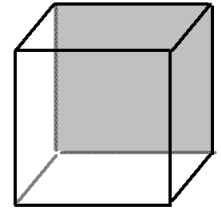
# Miller indices



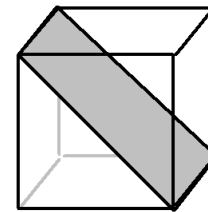
(001)



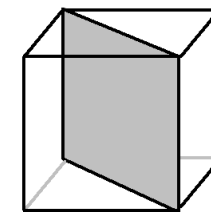
(100)



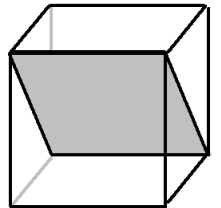
(010)



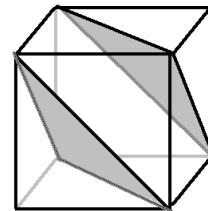
(101)



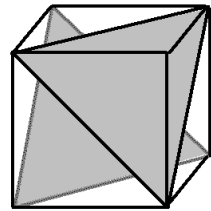
(110)



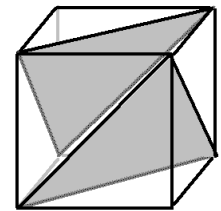
(011)



(111)



(1 $\bar{1}$ 1)

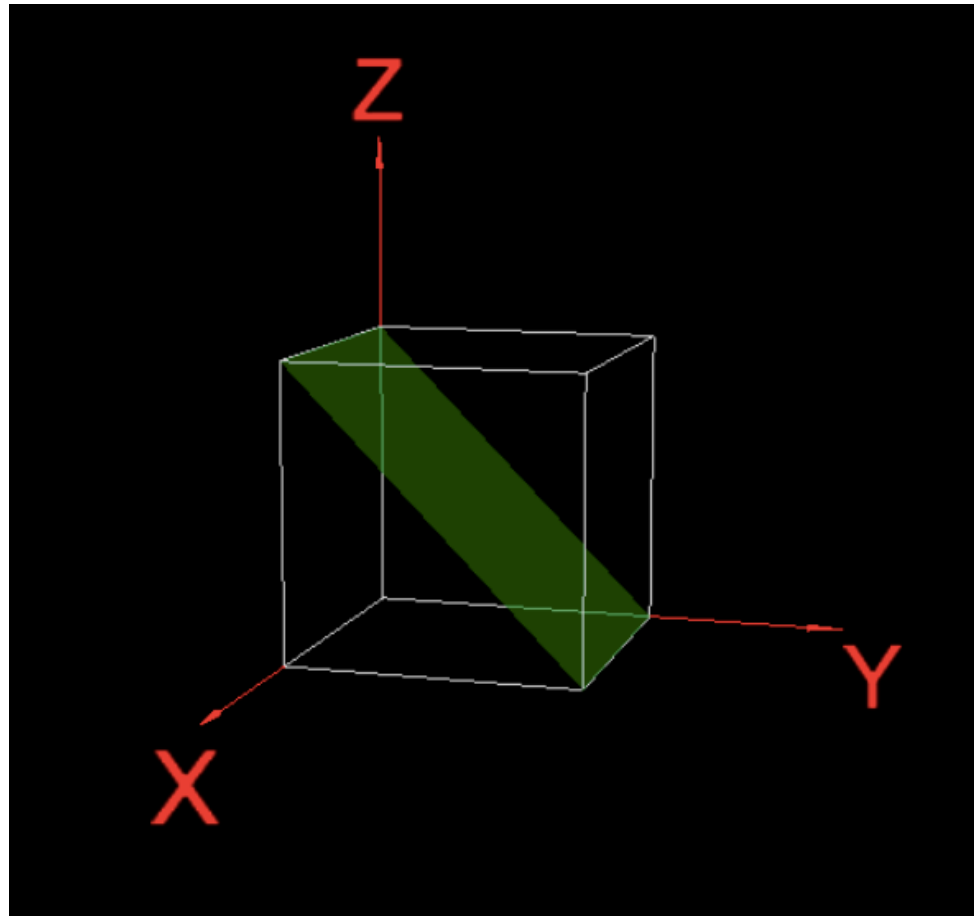


( $\bar{1}$ 11)

**[uvw] is a crystal direction,  
 <uvw> is a family of directions  
 (hkl) is a crystal plane,  
 {hkl} is a family of planes.**

# VMSE Screenshot - Crystallographic Planes

---



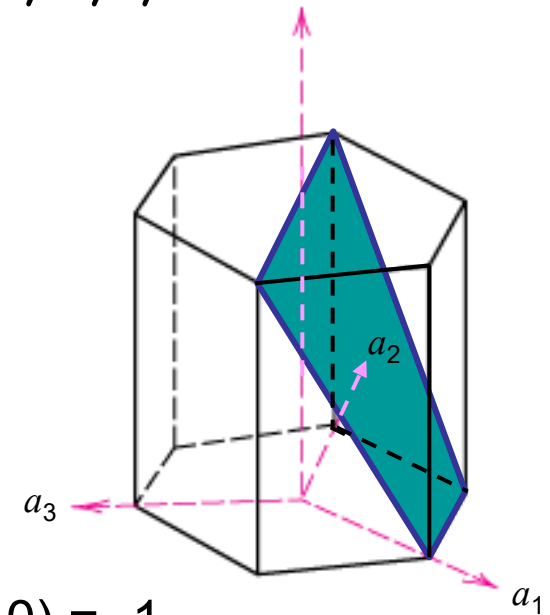
Additional practice on indexing crystallographic planes

# Crystallographic Planes (HCP)

- For hexagonal unit cells a similar procedure is used
  - Determine the intercepts with the  $a_1$ ,  $a_2$ , and  $z$  axes, then determine the Miller-Bravais Indices  $h$ ,  $k$ ,  $i$ , and  $l$

example

	$a_1$	$a_2$	$c$
1. Relocate origin – not needed			
2. Intercepts	$a$	$\infty a$	$c$
3. Reciprocals	$1/a$	$1/\infty a$	$1/c$
4. Normalize	$a/a$	$a/\infty a$	$c/c$
	1	0	1
5. Reduction	$h = 1$	$k = 0$	$l = 1$
6. Determine index $i = -(h + k)$			$i = -(1 + 0) = -1$
7. Miller-Bravais Indices	$(10\bar{1}1)$		

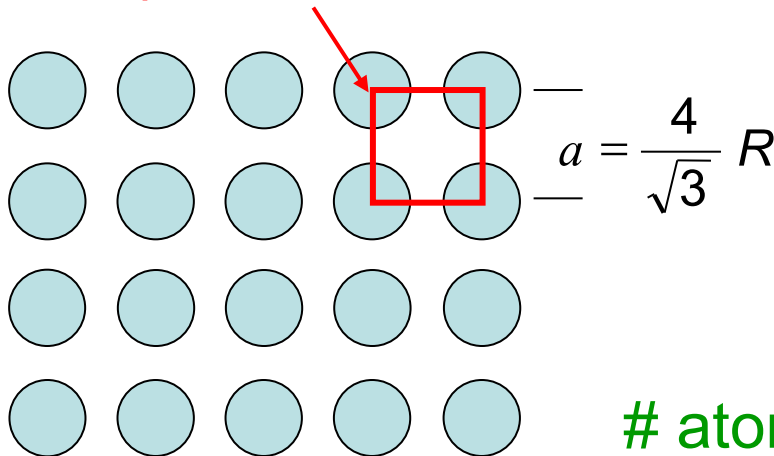


# Planar Density of Atoms (PD)

$$\text{PD} = \frac{\text{number of atoms centered on a plane}}{\text{area of plane}}$$

ex: planar density of (100) plane of BCC Fe

2D repeat unit



There are 4 quarter atoms  
= 1 atom centered on plane

$$a = \frac{4}{\sqrt{3}} R = \frac{4}{\sqrt{3}} (0.1241 \text{ nm}) = 0.287 \text{ nm}$$

Radius of iron,  
 $R = 0.1241 \text{ nm}$

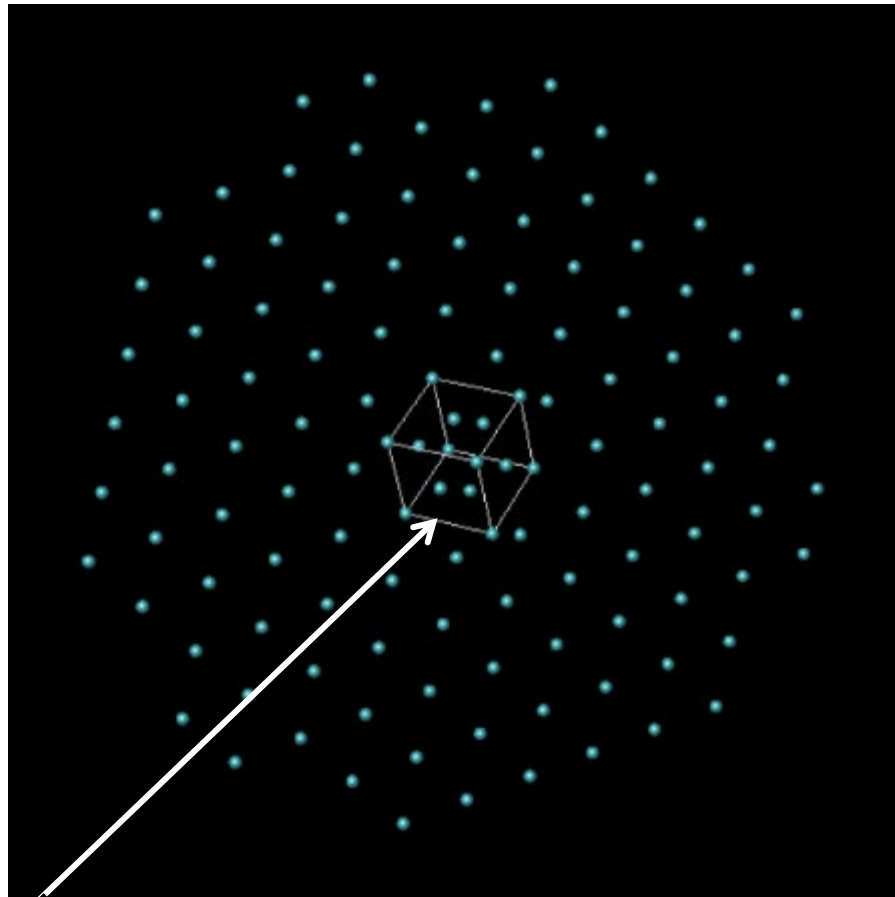
# atoms

$$\text{PD} = \frac{1}{a^2} = \frac{1 \text{ atom}}{(0.287 \text{ nm})^2} = 12.1 \frac{\text{atoms}}{\text{nm}^2}$$

area

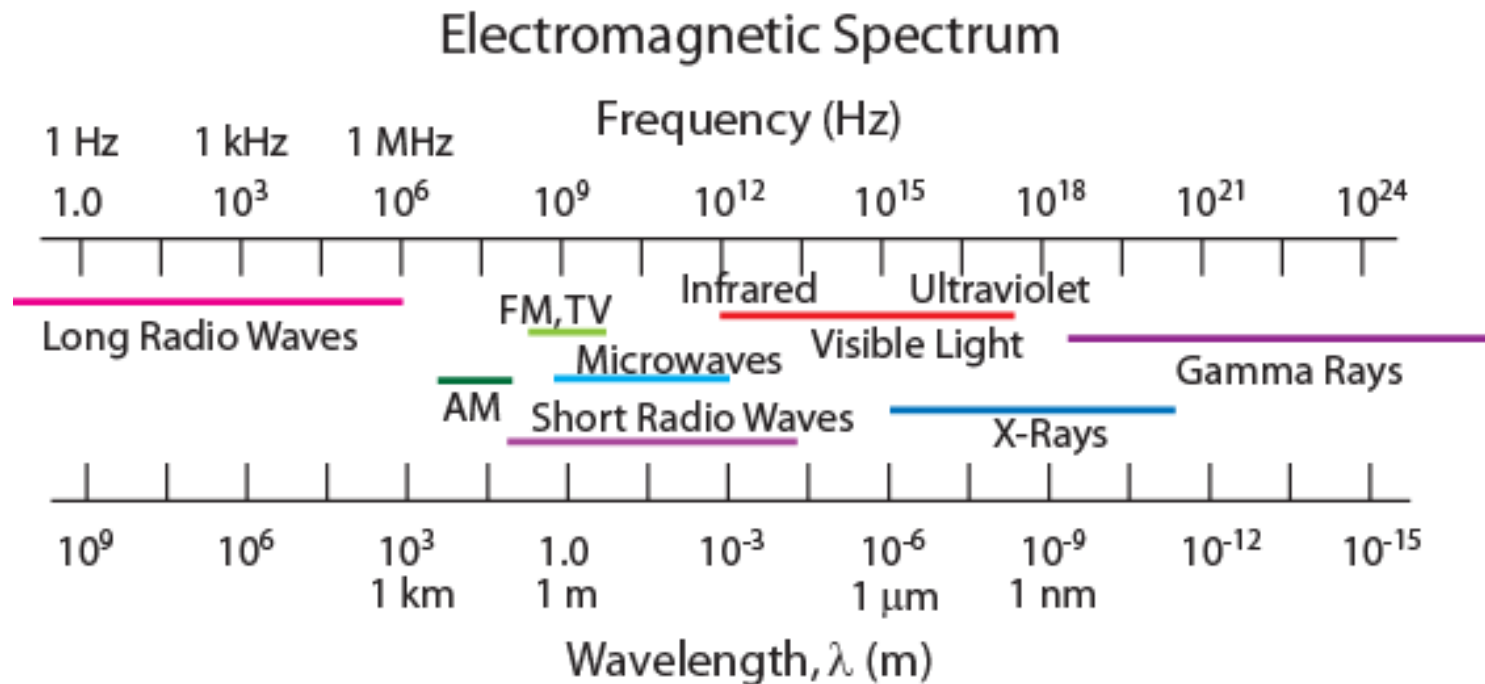
# VMSE Screenshot - Atomic Packing - (111) Plane for FCC

---



FCC Unit Cell

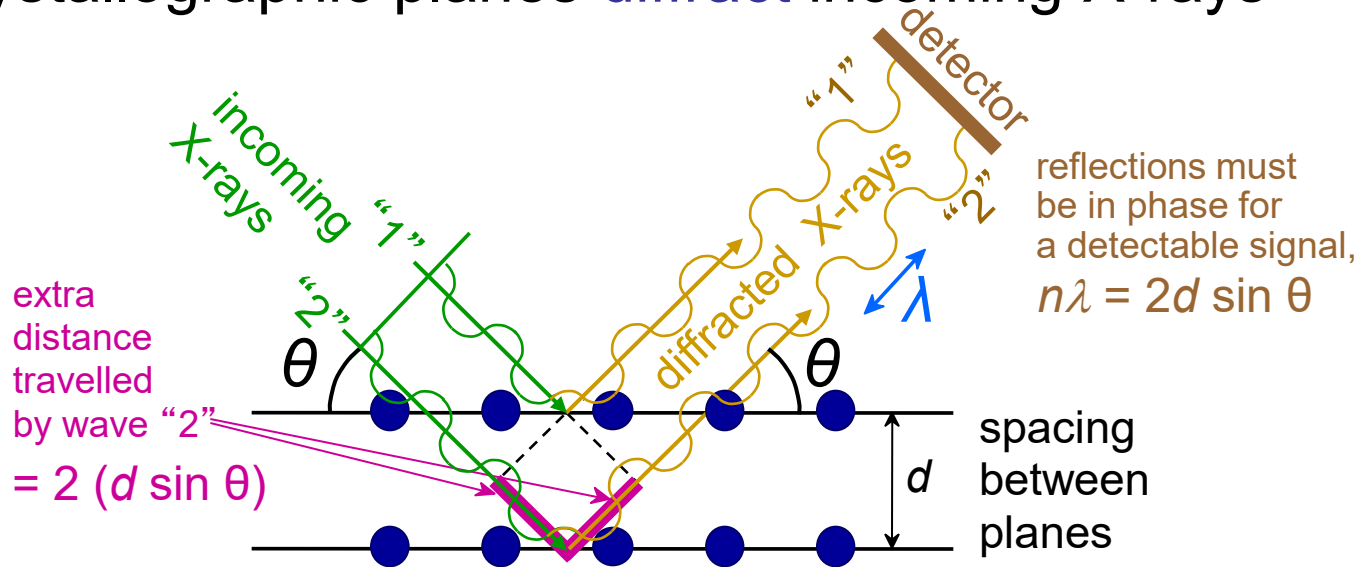
# X-Ray Diffraction



- To diffract light, the diffraction grating spacing must be comparable to the light wavelength.
- X-rays are diffracted by planes of atoms.
- Interplanar spacing is the distance between parallel planes of atoms.

# X-Rays to Determine Crystal Structure

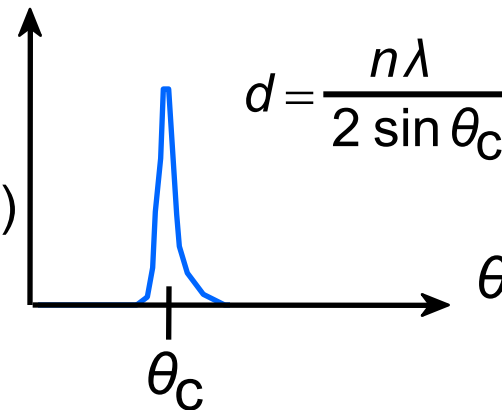
- Crystallographic planes **diffract** incoming X-rays



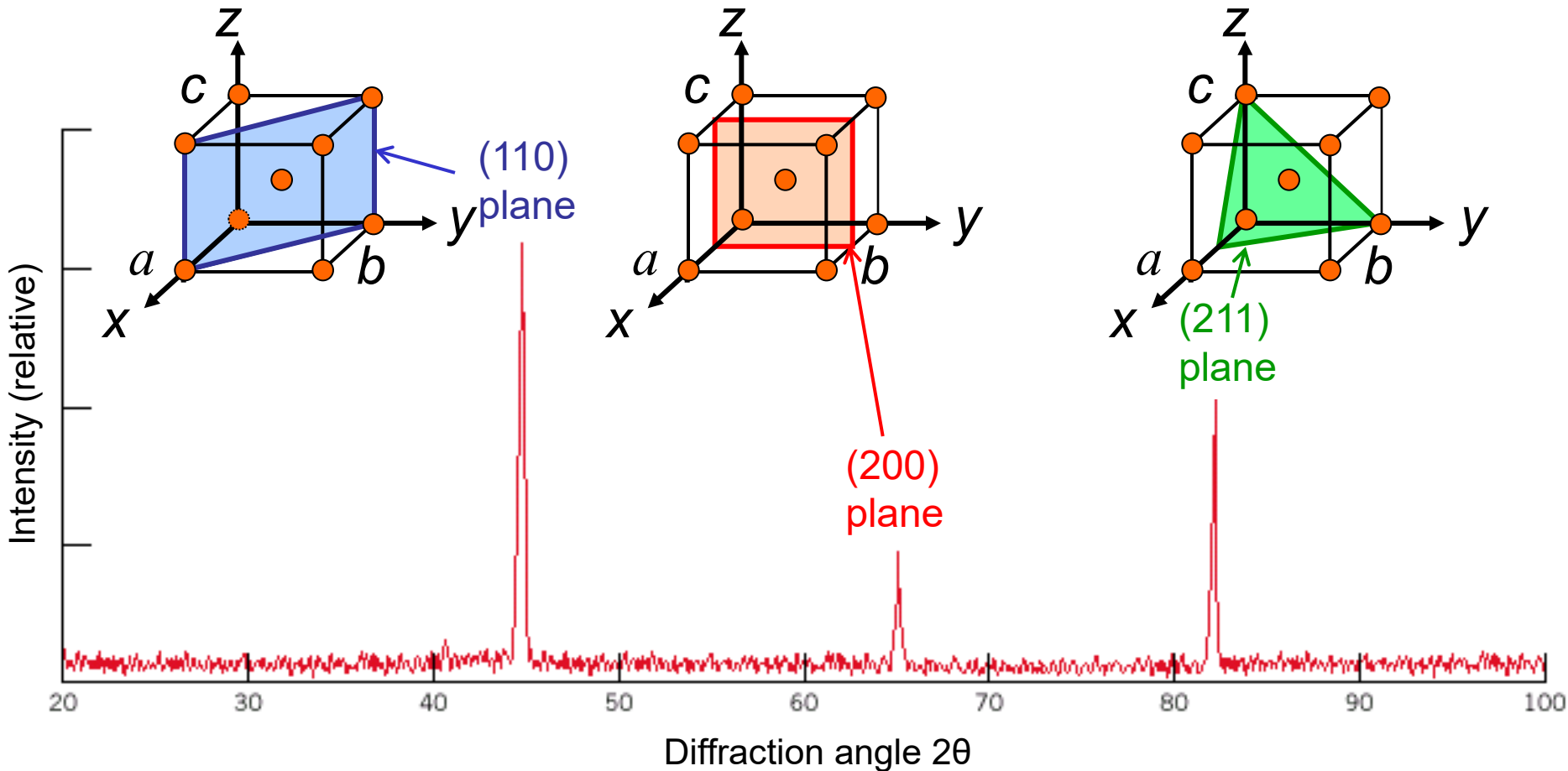
Measurement of diffraction angle,  $\theta_c$ , allows computation of interplanar spacing,  $d$ .

Diffraction occurs when  $\theta = \theta_c$

X-ray intensity (measured By detector)



# X-Ray Diffraction Pattern



Diffraction pattern for polycrystalline  $\alpha$ -iron (BCC)

# Summary

---

- Atoms may assemble into **crystalline** (ordered) or **amorphous** (disordered) structures.
- Common metallic crystal structures are **FCC**, **BCC**, and **HCP**. **Coordination number** and **atomic packing factor** are the same for both FCC and HCP crystal structures.
- We can calculate the theoretical **density** of a metal, given its **crystal structure**, **atomic weight**, and **unit cell lattice parameters**.
- **Crystallographic points**, **directions** and **planes** may be specified in terms of indexing schemes.
- **Atomic** and **planar densities** are related to crystallographic directions and planes, respectively.

## Summary (continued)

---

- Materials can exist as **single crystals** or **polycrystalline**.
- For most single crystals, properties vary with crystallographic orientation (i.e., are **anisotropic**).
- For polycrystalline materials having randomly oriented grains, properties are independent of crystallographic orientation (i.e., they are **isotropic**).
- Some materials can have more than one crystal structure. This is referred to as **polymorphism** (or **allotropy**).
- **X-ray diffraction** is used for crystal structure and **interplanar spacing** determinations.